Greetings!

Dear students, in the previous notes we learn about the introduction of set language, and its notations. Now we are going to learn about the ordered pair and Cartesian product.

ORDERED PAIR

- ✓ An ordered pair is a set of inputs and outputs and represents a relationship between the two values.
- ✓ In mathematics, an **ordered pair** (a, b) is a pair of objects.
- ✓ The order in which the objects appear in the pair is significant: the ordered pair (a, b) is different from the ordered pair (b, a) unless a = b.
- ✓ Let (a_1,b_1) and (a_2,b_2) be ordered pairs. Then the characteristic property of the ordered pair is : $(a_1,b_1) = (a_2,b_2)$ if and only if $a_1 = a_2$ and $b_1 = b_2$.

Example:

Observe the seating plan in an auditorium. To help orderly occupation of seats, tokens with numbers such as (1,5), (7,16), (3,4), (10,12) etc. are issued. The person who gets (4,10) will go to row 4 and occupy the 10th seat. Thus the first number denotes the row and the second number, the seat.

Which seat will the visitor with token (5,9) occupy? Can he go to 9th row and take the 5th seat? Do (9,5) and (5,9) refer to the same location? No, certainly!

What can you say about the tokens (2,3), (6,3) and (10,3)? This is one example where a pair of numbers, written in a particular order, precisely indicates a location. Such a number pair is called an ordered pair of numbers. This notion is skilfully used to mathematize the concept of a "Relation".

CARTESIAN PRODUCT

Definition

If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A$, $b \in B$ is called the Cartesian Product of A and B, and is denoted by A x B.

Thus A X B = { (a, b) / $a \in A$, $b \in B$}.

The set of all ordered pairs whose first entry is in some set A and whose second entry is in some set B is called the Cartesian product of A and B, and written A × B

The **"Cartesian product"** is also referred as **"cross Product"**

Illustration

Let us consider the following two sets.

A is the set of 3 vegetables and B is the set of 4 fruits. That is,

A = {carrot, brinjal, ladies finger} and B = {apple, orange, grapes, strawberry}

What are the possible ways of choosing a vegetable with a fruit?

Vegetables (A)	Fruits(B)
Carrot (c)	Apple (a)
Brinjal (b)	Orange (o)
Ladies finger (l)	Grpes (g)
	Straw berry(s)

We can select them in 12 distinct pairs as given below.

(c, a), (c, o), (c, g), (c, s), (b, a), (b, o), (b, g), (b, s), (l, a), (l, o), (l,g), (l, s)

This collection represents the Cartesian product of the set of vegetables and set of fruits.

Now we are enter into our book example

Example : 1.1

If $A = \{1,3,5\}$ and $B = \{2,3\}$ then (i) find $A \times B$ and $B \times A$.

(ii) Is $A \times B = B \times A$? If not why? (iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$

Solution:

Given that A = {1,3,5} and B = {2,3}

(i) $A×B$ = {1,3,5} × {2,3} = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)} ...(1)

$B×A$ = {2,3} × {1,3,5} = {(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)} ...(2)

(ii) From (1) and (2) we conclude that $A×B \neq BA$ *as (1,2)* $\neq$ *(2,1) and (1,3)* $\neq$*(3,1) etc*

(iii) $n(A)$=3; $n(B)$ = 2.

From (1) and (2) we observe that, $n(A×B) = n(B×A) = 6$;

we see that, $n(A) × n(B)$ = 3 × 2 = 6 and $n(B) × n(A)$ = 2×3 = 6

Hence, $n(A×B) = n(B×A) = n(A) × n(B) = 6$.

Thus, $n(A×B) = n(B×A) = n(A) × n(B)$.

Example: 1.2

If A x B = { (3,2) , (3,4) , (5,2) ,(5, 4) } then find **A** and **B .**

Solution

A x B = { (3,2) , (3,4) , (5,2) ,(5, 4) }

We have A = { set of all first coordinates of elements of A x B }

A = { 3,5 }

B = { set of all first coordinates of elements of A x B }

B = { 2,4 }

Thus A = { 3,5 } and B = { 2,4 }

Example 1.3 Let $A = \{x \in \mathbb{N} \mid 1 < x < 4\}$, $B = \{x \in \mathbb{W} \mid 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} \mid x < 3\}$. Then verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution $A = \{x \in \mathbb{N} \mid 1 < x < 4\} = \{2, 3\}$, $B = \{x \in \mathbb{W} \mid 0 \leq x < 2\} = \{0,1\}$,
$C = \{x \in \mathbb{N} \mid x < 3\} = \{1,2\}$

(i) $A\times(B\cup C) = (A\times B)\cup(A\times C)$

$B\cup C = \{0,1\}\cup\{1,2\} = \{0,1,2\}$

$A\times(B\cup C) = \{2,3\}\times\{0,1,2\} = \{(2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\}$ ...(1)

$A\times B = \{2,3\}\times\{0,1\} = \{(2,0),(2,1),(3,0),(3,1)\}$

$A\times C = \{2,3\}\times\{1,2\} = \{(2,1),(2,2),(3,1),(3,2)\}$

$(A\times B)\cup(A\times C) = \{(2,0),(2,1),(3,0),(3,1)\}\cup\{(2,1),(2,2),(3,1),(3,2)\}$

$= \{(2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\}$ ...(2)

From (1) and (2), $A\times(B\cup C) = (A\times B)\cup(A\times C)$ is verified.

(ii) $A\times(B\cap C) = (A\times B)\cap(A\times C)$

$B\cap C = \{0,1\}\cap\{1,2\} = \{1\}$

$A\times(B\cap C) = \{2,3\}\times\{1\} = \{(2,1),(3,1)\}$ ... (3)

$A\times B = \{2,3\}\times\{0,1\} = \{(2,0),(2,1),(3,0),(3,1)\}$

$A\times C = \{2,3\}\times\{1,2\} = \{(2,1),(2,2),(3,1),(3,2)\}$

$(A\times B)\cap(A\times C) = \{(2,0),(2,1),(3,0),(3,1)\}\cap\{(2,1),(2,2),(3,1),(3,2)\}$

$= \{(2,1),(3,1)\}$ ... (4)

From (3) and (4), $A\times(B\cap C) = (A\times B)\cap(A\times C)$ is verified.

Now we enter into exercise problems

Exercise 1.1 (Sums from 1 to 3)

Question : 1

Find A × B, A × A and B × A

(i) A = {2,-2,3} and B = {1,-4}

(ii) A = B = {p,q]

(iii) A= {m,n} ; B = (Φ)

Solution:

(i) A = {2,-2,3}, B = {1,-4}

A × B = {(2, 1), (2, -4), (-2, 1), (-2, -4), (3,1) , (3,-4)}

A × A = {(2, 2), (2,-2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3,3) }

B × A = {(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4,3)}

(ii) A = B = {(p,q)]

A × B = {(p, p), {p, q), (q, p), (q, q)}

A × A = {(p, p), (p, q), (q, p), (q, q)}

B × A = {(p,p), {p, q), (q, p), (q, q)}

(iii) A = {m,n} × Φ

A × B = { }

A × A = {(m, m), (m, n), (n, m), (n, n)}

B × A = { }

Question : 2

Let A= {1,2,3} and B = {× | x is a prime number less than 10}. Find A × B and B × A.

Answer:

A = {1,2,3}, B = {2, 3, 5, 7}

A × B = {1,2,3} × {2, 3, 5, 7}

= {(1, 2) (1, 3) (1, 5) (1, 7) (2, 2)

(2, 3) (2, 5) (2, 7)(3, 2) (3, 3) (3, 5) (3, 7)}

B × A = {2, 3, 5, 7} × {1,2,3}

= {(2, 1)(2, 2)(2, 3)(3, 1)(3, 2)(3, 3) (5, 1)(5, 2)(5, 3) (7, 1) (7,2)(7, 3)}

Question : 3

If B × A = {(-2, 3),(-2, 4),(0, 3),(0, 4),(3, 3), (3, 4)} find A and B.

Solution:

B × A ={(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)}

A = {3, 4), B = { -2, 0, 3}

@@@@ Thank you @@@@