VINAYAKAR MATRICULATION HIGHER SECONDARY SCHOOL, SIVAKASI.

10^{th} Standard – MATHS

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Greetings,

Dear students in the previous notes we learn about Special Series examples and their exercise Problems. Now we are learn more about the concept of **ARITHMETIC PROGRESSION.**

INTRODUCTION OF ARITHMETIC PROGRESSION:

Arithmetic Progression:

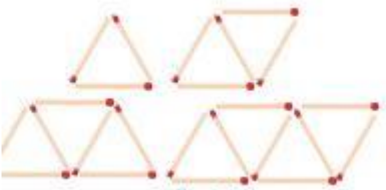
Any sequence in which the difference between every successive term is constant, and then it is called 'Arithmetic sequence'. It would be in ascending or descending form according to the constant number.

Example: 0,3, 6, 9, 12

Illustration 1

Make the following figure using match sticks

i) How many match sticks are required for each figure? 3, 5, 7 and 9



ii) Can we find the difference between the successive numbers?

5-3= 7-5 = 9-7 =2

Therefore the difference between successive numbers is always 2.

Illustration 2

A man got a job whose initial monthly salary is fixed at Rs. 10,000 with an annual increment of Rs.2000. His salary during 1^{st}, 2^{nd} and 3^{rd} years will be Rs. 10000 , Rs.12000, and RS.14000 respectively.

- If we calculate the difference of the salaries for the successive years
  - We get, 12000 - 10000 = 2000
    14000 - 12000 = 2000
- Thus the difference between the successive numbers is always 2000.
- From the above we notice that the difference between successive terms always remains constant. This is called 'common difference'.

Definition:

Let 'a' and 'd' be real numbers. Thus the numbers form **a**, **a+d**, **a+2d**, **a+3d, a + 4d …** is said to form **"Arithmetic Progression"** denoted by A.P.

The number "a" is called the **first term** and 'd' is called the **common difference**.

(For Ex). 2,4,6,8,10,12, … an A.P

whose first term is a = 2

common difference (d) = 4-2= 2

Note:

- The difference between any two successive terms of an A.P is always constant. That is called "common difference".
- If there are finite numbers of terms in an A.P then it is called Finite Arithmetic Progression.
- If there are infinitely many terms in an A.P. then it is called Infinite Arithmetic Progression.

Terms and Common Difference of an A.P.

1. The terms of an A.P. can be written as

$t_1 = a = a + (1-1)d$, $t_2 = a + d = a + (2-1)d$,

$t_3 = a + 2d = a + (3-1)d$, $t_4 = a + 3d = a + (4-1)d$, . . .

In general, the n^{th} term denoted by t_n can be written as $t_n = a + (n-1)d$.

In an AP, n^{th} term is, $t_n = a + (n-1)d$, here, a is the first term, d is the common difference.

2. In general to find the common difference of an A.P we should subtract first term from the second term, second term from the third term and so on.

(For Ex) $t_1 = a$

$t_2 = a+d$

Therefore $t_2 - t_1 = (a+d) - a = d$

Similarly, $t_2 = a+d$, $t_3 = a+2d$

$t_3 - t_2 = (a+2d) - (a+d) = d$

In general $d = t_2 - t_1 = t_3 - t_2 = \ldots$

$d = t_n - t_{n-1}$ for n = 2, 3, 4 …

An Arithmetic progression having a common difference of zero is called a constant arithmetic progression.

In a finite A.P. whose first term is a and last term l, then the number of terms in the A.P. is given by $l = a + (n-1)d \Rightarrow n = \left(\frac{l-a}{d}\right) + 1$

Example 2.23 Check whether the following sequences are in A.P. or not?

(i) $x+2,\ 2x+3,\ 3x+4,\ldots$ (ii) $2, 4, 8, 16,\ldots.$ (iii) $3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2},\ldots$

Solution To check that the given sequence is in A.P., it is enough to check if the differences between the consecutive terms are equal or not.

(i) $t_2 - t_1 = (2x+3)-(x+2) = x+1$

$t_3 - t_2 = (3x+4)-(2x+3) = x+1$

$t_2 - t_1 = t_3 - t_2$

Thus, the differences between consecutive terms are equal.

Hence the sequence $x+2, 2x+3, 3x+4,\ldots$ is in A.P.

(ii) $t_2 - t_1 = 4-2 = 2$

$t_3 - t_2 = 8-4 = 4$

$t_2 - t_1 \neq t_3 - t_2$

Thus, the differences between consecutive terms are not equal. Hence the terms of the sequence 2, 4, 8, 16, . . . are not in A.P.

(iii) $t_2 - t_1 = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

$t_3 - t_2 = 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$

$t_4 - t_3 = 9\sqrt{2} - 7\sqrt{2} = 2\sqrt{2}$

Thus, the differences between consecutive terms are equal. Hence the terms of the sequence $3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2},\ldots$ are in A.P.

Example 2.24 Write an A.P. whose first term is 20 and common difference is 8.

Solution First term $= a = 20$; common difference $= d = 8$

Arithmetic Progression is $a,\ a+d,\ a+2d,\ a+3d,\ldots$

In this case, we get $20,\ 20+8,\ 20+2(8),\ 20+3(8),\ldots$

So, the required A.P. is 20, 28, 36, 44, …

Example 2.25 Find the 15^{th}, 24^{th} and n^{th} term (general term) of an A.P. given by 3, 15, 27, 39,…

Solution We have, first term $= a = 3$ and common difference $= d = 15-3 = 12$.

We know that n^{th} term (general term) of an A.P. with first term a and common difference d is given by $t_n = a+(n-1)d$

$$t_{15} = a + (15-1)d = a + 14d = 3 + 14(12) = 171$$

(Here $a = 3$ and $d = 12$)

$$t_{24} = a + (24-1)d = a + 23d = 3 + 23(12) = 279$$

The n^{th} (general term) term is given by $t_n = a + (n-1)d$

Thus, $t_n = 3 + (n-1)12$

$t_n = 12n - 9$

Example 2.26 Find the number of terms in the A.P. 3, 6, 9, 12,…, 111.

Solution

First term $a = 3$; common difference $d = 6 - 3 = 3$; last term $l = 111$

We know that, $n = \left(\frac{l-a}{d}\right) + 1$

$$n = \left(\frac{111-3}{3}\right) + 1 = 37$$

Thus the A.P. contain 37 terms.

Example 2.27 Determine the general term of an A.P. whose 7^{th} term is -1 and 16^{th} term is 17.

Solution Let the A.P. be $t_1, t_2, t_3, t_4, \ldots$

It is given that $t_7 = -1$ and $t_{16} = 17$

$a + (7-1)d = -1$ and $a + (16-1)d = 17$

$a + 6d = -1$ ...(1)

$a + 15d = 17$ ...(2)

Subtracting equation (1) from equation (2), we get $9d = 18 \Rightarrow d = 2$

Putting $d = 2$ in equation (1), we get $a + 12 = -1 \therefore a = -13$

Hence, general term $t_n = a + (n-1)d$

$= -13 + (n-1) \times 2 = 2n - 15$

NOW, LET US SEE THE EXERCISE 2.5 (1- 8) SUMS

Question 1

Check whether the following sequences are in A.P.

(i) a – 3, a – 5, a – 7, ... (ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \ldots\ldots$ (iii) 9, 13, 17, 21, 25 ...

(iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots\ldots$ (v) 1,-1, 1,-1, 1,-1,...

Solution:

To prove it is an A.P, we have to show $d = t_2 - t_1 = t_3 - t_2$.

(i) a – 3, a – 5, a – 7.........

$t_1 = a - 3$, $t_2 = a - 5$,$t_3 = a - 7$

$d = t_2 - t_1 = a - 5 - (a - 3) = a - 5 - a + 3 = -2$

$\therefore d = -2$

$d = t_3 - t_2 = a - 7 - (a - 5) = a - 7 - a + 5 = -2$

Here, $t_2 - t_1 = t_3 - t_2 = -2$

Hence is an A.P.

(ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \ldots\ldots$

$d = t_2 - t_1$

$\Rightarrow \frac{1}{3} - \frac{1}{2}$

$= \frac{2-3}{6}$

$= \frac{-1}{6}$

$d = t_3 - t_2$

$\frac{1}{4} - \frac{1}{3}$

$= \frac{3-4}{12} = \frac{-1}{12}$

$\frac{-1}{6} \neq \frac{-1}{12}$

$\Rightarrow t_2 - t_1 \neq t_3 - t_2$ $\therefore$ It is not an A.P.

(iii) 9, 13, 17, 21, 25, . . .

$d = t_2 - t_1 = 13 - 9 = 4$

$d = t_3 - t_2 = 17 - 13 = 4$

$4 = 4$

$\therefore$ It is an A.P.

(iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$

$d = t_2 - t_1 = 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$

$d = t_3 - t_2 = \frac{1}{3} - 0 = \frac{1}{3}$

$\frac{1}{3} = \frac{1}{3}$

$\therefore$ It is an A.P.

(v) 1,-1, 1,-1, 1,-1,...

$d = t_2 - t_1 = -1 - 1 = -2$

$d = t_3 - t_2 = 1 - (-1) = 2$

$-2 \neq 2 \therefore$ It is not an A.P.

Question 2

First term 'a' and common difference 'd' are given below. Find the corresponding A.P.

(i) a = 5, d = 6 (ii) a = 7, d = 5 (iii) a = 34, d = 12

Solution:

(i) a = 5, d = 6

A.P = a, a + d, a + 2d, ...

= 5, 5 + 6, 5 + (2 × 6), ...

= 5, 11, 17,...

(ii) a = 7,d = -5

A.P. = a, a + d, a + 2d,...

= 7, 7 + (-5), 7 + 2(-5), ...

= 7, 2, -3, ...,...

(iii) $a = \frac{3}{4}$, $d = \frac{1}{2}$

$$A.P = a, a+d, a+2d, \ldots$$

$$= \frac{3}{4}, \frac{3}{4}+\frac{1}{2}, \frac{3}{4}+2\left(\frac{1}{2}\right), \ldots$$

$$= \frac{3}{4}, \frac{3+2}{4}, \frac{3+4}{4}, \ldots$$

$$A.P = \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots.$$

Question 3

Find the first term and common difference of the Arithmetic Progressions whose n^{th} terms are given below (i) $t_n = -3 + 2n$ (ii) $t_n = 4 - 7n$

Solution:

(i) $t_n = -3 + 2n$

$t_1 = -3 + 2(1) = -3 + 2 = -1$

$t_2 = -3 + 2(2) = -3 + 4 = 1$

First term (a) = -1 and

d = 1 – (-1) = 1 + 1 = 2

(ii) $t_n = 4 - 7n$

$t_1 = 4 - 7(1) = 4 - 7 = -3$

$t_2 = 4 - 7(2) = 4 - 14 = -10$

First term (a) = – 3 and

d = 10 – (-3) = – 10 + 3= – 7

Question 4

Find the 19th term of an A.P. -11, -15, -19, ……….

Solution:

A.P = -11, -15, -19, ……..

a = -11 $d = t_2 - t_1$ =-15-(-11)   n = 19

= -15 + 11

= -4

∴ $t_n = a + (n-1)d$

t_{19} = -11 + (19 – 1)(-4)

= -11 + 18 × -4

= -11 – 72

= -83

Question 5.

Which term of an A.P. 16, 11, 6, 1,… is -54?

Solution:

First term (a) = 16   Common difference (d) = 11 – 16 = -5

t_n = – 54

a + (n – 1) d = -54

16 + (n – 1) (-5) = -54

54 + 21 = -54

54 + 21 = 5n

75 = 5n

$n = \frac{75}{5} = 15$

The 15th term is – 54

Question 6.

Find the middle term(s) of an A.P. 9, 15, 21, 27, ……. ,183.

Solution:

A.P = 9, 15, 21, 27,…, 183

No. of terms in an A.P. is

$$n = \left(\frac{l-a}{d}\right) + 1$$

a = 9, l = 183, d = 15 – 9 = 6

$n = \left(\frac{183-9}{6}\right) + 1$

$= \left(\frac{174}{6}\right) + 1$

= 29 + 1 = 30

∴ No. of terms = 30. The middle must be 15th term and 16th term.

∴ $t_{15} = a + (n-1)d$	$t_{16} = a + 15d$
= 9 + 14 × 6	= 9 + 15 × 6
=9 + 84	= 9 + 90
= 93	= 99

∴ The middle terms are 93, 99.

Question 7

If nine times the ninth term is equal to the fifteen times fifteenth term, Show that six times twenty fourth term is zero.

Solution:

$t_n = a + (n-1)d$

9 times 9^{th} term = 15 times 15^{th} term

$9t_9 = 15\, t_{15}$

9[a + 8d] = 15[a + 14d]

9a + 72d = 15a + 210d

9a – 15a + 72 d – 210 d = 0

-6a – 138 d = 0

6a + 138 d = 0

6 [a + 23 d] = 0

6 [a + (24 – 1)d] = 0

6 t_{24} = 0

∴ Six times 24th terms is 0.

Question 8.

If 3 + k, 18 – k, 5k + 1 are in A.P. then find k.

Solution:

3 + k, 18 – k, 5k + 1 are in A.P

⇒ 2b = a + c if a, b, c are in A.P

$$\therefore \underbrace{3+k}_{a}, \underbrace{18-k}_{b}, \underbrace{5k+1}_{c}$$

$$\begin{aligned} 2b &= a+c \\ \Rightarrow \quad 2(18-k) &= 3+k+5k+1 \\ 36-2k &= 4+6k. \\ 6k+2k &= 36-4 \\ 8k &= 32 \\ k &= \frac{32}{8} = 4 \end{aligned}$$

@@@@ Thank You @@@@