Matrices and Determinants

In Mathematics, one of the interesting, easiest and important topic is Matrices and Determinants. Every year you will get at least 1 - 3 questions in JEE Main and other exams, directly and indirectly, the concept of this chapter will be involved in many other chapters, like integral and differential calculus. Concept of this chapter will be used for the axis-transformation concept. This chapter is totally new from the student point of view as you will see this chapter directly in 12th. So some students may find Matrices and Determinant little challenging to understand and solve problems initially. But as you solve more and more problems in this chapter, you will get familiar with concepts and chapter as a whole, then you will find that this is one of the easiest chapters. Afterward, the questions will appear easy for you. Matrices part may seem a little more difficult than Determinant but in the end, you will find both are easy to grasp,

Why Matrices and Determinants:

Matrices and Determinant find wide ranges of application in real-life problem,

For example in adobe Photoshop software matrix are used to process linear transformation to render images.

- A square matrix is used to represent a linear transformation of a geometric object.
- In computer programming matrices and its inverse are used for encrypting messages, to store data, perform queries and used as a data structure to solve algorithmic problems, etc.
- In robotics, the movement of the robot is programmed using a calculation based on matrices.

After studying this chapter,

1. It will be easy for you to understand the concept of the array in computer science (if u have taken computer science in +2).

2. It will be helping you to solve the problem involving simultaneous equation with as many unknown variables as equations.

3. Determinant will help you to solve problems related to areas and volume like the area of triangle and volume of a tetrahedron.

4. It will be helping you to organize your work in a much better way in the form of matrices and hence will help you to be clear in your mind in daily life.

5. And obviously, the chapter itself will help you to score some marks in the exam as it gets about 7% weight in JEE main and around similar weight in other exams.

Matrix: Set of numbers or objects or symbols represented in the form of the rectangular array is called a matrix.

The order of the matrix is defined by the number of rows and number of columns present in the rectangular array of representation.

For example

Matrix $\begin{bmatrix} 2 & 4 & -3 \\ 5 & 4 & 6 \end{bmatrix}$ has 2 rows and 3 columns so its order is said to be 2 × 3.

Any general element of the matrix is represented by a_{ij}, where a_{ij} represents the elements of the i^{th} row and j^{th} column.

There are different types of Matrices. Here they are -

1) Row matrix
2) Column matrix
3) Null matrix
4) Square matrix
5) Diagonal matrix
6) Upper triangular matrix
7) Lower triangular matrix
8) Symmetric matrix
9) Skew -symmetric matrix
10) Horizontal matrix
11) Vertical matrix
12) Identity matrix

1. What is a Null Matrix?

If in a matrix all the elements are zero then it is called a zero matrix and it is generally denoted by 0. Thus, A = $[a_{ij}]$ m x n is a zero-matrix if a_{ij}= 0 for all i and j.

The first matrix O is a 2 × 2 matrix with all the elements equal to zero and the second matrix O is a 3 × 3 matrix with all the elements equal to zero.

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. What is a Triangular Matrix?

A square matrix is known to be triangular if all of its elements above the principal diagonal are zero then the triangular matrix is known as a lower triangular matrix or all of its elements below the principal diagonal are zero then the triangular matrix is known as upper triangular matrix).

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}$$

The matrix given above is a 3 × 3 upper triangular matrix.

The matrix given below is an example of a 3 × 3 lower triangular matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 6 & 5 \\ 0 & 0 & 9 \end{bmatrix}$$

3. What is a Vertical Matrix?

A matrix of order m x n is known as a vertical matrix if m > n, where m is equal to the number of rows and n is equal to the number of columns.

Matrix Example

$$\begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}$$

In matrix example given below the number of rows (m) = 4 , whereas the number of columns (n) = 2. Therefore, this makes the matrix a vertical matrix.

4. What is a Horizontal Matrix?

A matrix of order m x n is known as a horizontal matrix if n > m, where m is equal to the number of rows and n is equal to the number of columns.

Matrix Example

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 1 \end{bmatrix}$$

In the matrix example given below the number of rows (m) = 2 , whereas the number of columns (n) = 4. Therefore, we can say that the matrix is a horizontal matrix.

5. What is a Row Matrix?

A matrix that has only one row is known as a row matrix. Thus A = $[a_{ij}]$ m x n is a row matrix if m is equal to 1. So, a row matrix can be represented as A = $[a_{ij}]$ 1 × n. It is known so because it has only one row and the order of a row matrix will hence always be equal to 1 × n.

Example of a Row matrix,

$$A = \begin{bmatrix} 4 & 6 & 9 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 & 9 & 2 & 5 \end{bmatrix}$$

In matrix example given above, matrix A has only one row and so matrix B has one row, therefore both matrices A and B are row matrices.

6. What is a Column Matrix?

A matrix that has one column is known as a Column matrix. Thus A = $[a_{ij}]$m x n is a column matrix if n is equal to 1. So, a row matrix can be represented as A = $[a_{ij}]$m × 1. It is known so because it has only one column and the order of a column matrix will hence always be equal to m × 1.

Example of a Column matrix,

$$A = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 9 \\ 8 \\ 2 \end{bmatrix}$$

In matrix example given above, matrix A has only one column and matrix B has one column, therefore both matrices A and B are column matrices.

7. What is a Diagonal Matrix?

If all the elements of the matrix, except the principal diagonal in any given square matrix, is equal to zero, it is known as a diagonal matrix. Thus,a square matrix A = $[a_{ij}]$ is a diagonal matrix if a_{ij} = 0, when i is not equal to j.

For example,

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

The example given above is a diagonal matrix as it has elements only in its diagonal.

8. What is a Symmetric Matrix?

A square matrix A = $[a_{ij}]$ is known as a Symmetric matrix if a_{ij} = a_{ji}, for all i,j values.

For example,

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 2 \end{pmatrix}$$

9. What is the Skew -Symmetric Matrix?

A square matrix A = $[a_{ij}]$ is a skew-symmetric matrix if a_{ij} = a_{ji}, for all values of i,j. Thus, in a skew-symmetric matrix all diagonal elements are equal to zero.

For example,

For example,

$$\begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}$$

10. What is an Identity Matrix?

If all the elements of a principal diagonal in a diagonal matrix are 1, then it is called a unit matrix. A unit matrix of order n can be denoted by In. Thus, a square matrix A = [a_{ij}] m × n is an identity matrix if all its diagonals have value 1.

For example,A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Operations on Matrices

Addition, subtraction and multiplication are the basic operations on the matrix. To add or subtract matrices, these must be of identical order and for multiplication; the number of columns in the first matrix equals the number of rows in the second matrix.

- Addition of Matrices
- Subtraction of Matrices
- Scalar Multiplication of Matrices
- Multiplication of Matrices

Addition of Matrices

If $A[a_{ij}]_{mxn}$ and $B[b_{ij}]_{mxn}$ are two matrices of the same order then their sum A + B is a matrix, and each element of that matrix is the sum of the corresponding elements. i.e. $A + B = [a_{ij} + b_{ij}]_{mxn}$

Consider the two matrices A & B of order 2 x 2. Then the sum is given by:

$$\begin{bmatrix} a1 & b1 \\ c1 & d1 \end{bmatrix} + \begin{bmatrix} a2 & b2 \\ c2 & d2 \end{bmatrix} = \begin{bmatrix} a1+a2 & b1+b2 \\ c1+c2 & d1+d2 \end{bmatrix}$$

Subtraction of Matrices

If A and B are two matrices of the same order, then we define $A - B = A + (-B)$.

Consider the two matrices A & B of order 2 x 2. Then the difference is given by:

$$\begin{bmatrix} a1 & b1 \\ c1 & d1 \end{bmatrix} - \begin{bmatrix} a2 & b2 \\ c2 & d2 \end{bmatrix} = \begin{bmatrix} a1-a2 & b1-b2 \\ c1-c2 & d1-d2 \end{bmatrix}$$

We can subtract the matrices by subtracting each element of one matrix from the corresponding element of the second matrix. i.e. $A - B = [a_{ij} - b_{ij}]_{mxn}$

Finding the Product of Two Matrices

In addition to multiplying a matrix by a scalar, we can multiply two matrices. Finding the **product of two matrices** is only possible when the inner dimensions are the same, meaning that the number of columns of the first matrix is equal to the number of rows of the second matrix. If A is an $m \times r$ matrix and B is an $r \times n$ matrix, then the product matrix AB is an $m \times n$ matrix.

Transpose of the matrix: If A is a matrix then the matrix obtained by changing the columns of a matrix with rows or rows with columns is called the transpose of the matrix.

Determinant

The determinant helps us find the inverse of a matrix, tells us things about the matrix that are useful in systems of linear equations, calculus and more.

Symbol

The **symbol** for determinant is two vertical lines either side.

Example: **|A|** means the determinant of the matrix **A**

Calculating the Determinant

First of all the matrix must be **square** (i.e. have the same number of rows as columns). Then it is just basic arithmetic. Here is how:**For a 2×2 Matrix**

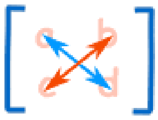
For a 2×2 matrix (2 rows and 2 columns):

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The determinant is:$|A| = ad - bc$

"The determinant of A equals a times d minus b times c"

It is easy to remember when you think of a cross:



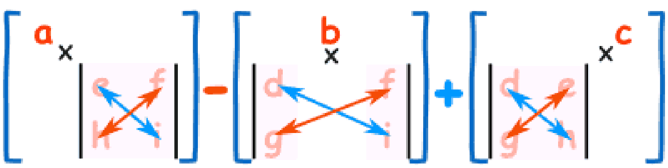
- Blue is positive (+ad),
- Red is negative (−bc)

For a 3×3 Matrix : For a 3×3 matrix (3 rows and 3 columns):

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The determinant is:$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$

"The determinant of A equals ... etc".It may look complicated, but **there is a pattern**:



To work out the determinant of a **3×3** matrix:

- Multiply **a** by the **determinant of the 2×2 matrix** that is **not in a**'s row or column.
- Likewise for **b**, and for **c**

- Sum them up, but remember the minus in front of the **b**

As a formula *(remember the vertical bars || mean "determinant of")*:

$$|A| = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \cdot \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \cdot \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

"The determinant of A equals a times the determinant of ... etc"

Steps to Finding Each Minor Of A Matrix:

1. Delete the ith row and jth column of the matrix.

2. Compute the determinant of the remaining matrix after deleting the row and column of step 1.

Cofactors: To find the cofactors of a matrix, just use the minors and apply the following formula: $C_{ij} = (-1)^{i+j}$ Mij where M_{ij} is the minor in the i^{th} row, j^{th} position of the matrix.

Transpose of a Determinant: The transpose of a determinant is a determinant obtained after interchanging the rows and columns.

D = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D^T = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Symmetric, Skew symmetric, Asymmetric Determinants:

(i) A determinant is symmetric if it is identical to its transpose. The i^{th} row is identical to its i^{th} column. I.e. $a_{ij} = a_{ji}$ for all values of i and j.

(ii) A determinant is skew symmetric if it is identical to its transpose having the sign of each element inverted. I.e. $a_{ij} = -a_{ji}$ for all values of i and j.

(iii) A determinant is asymmetric if it is neither symmetric nor skew symmetric.

Properties of determinants:

(i) D = D'

(ii) If a determinant has all the elements zero in any row or column, then D = 0

(iii) If any two rows or columns of a determinant be interchanged, then D' = -D.

(iv) If a determinant has any two rows or columns identical, then D = 0.

(v) If all the elements of any row or column be multiplied by the same number k, then D' = kD.

(vi) If each element of any row or column can be expressed as a sum of two terms, then the determinant can be