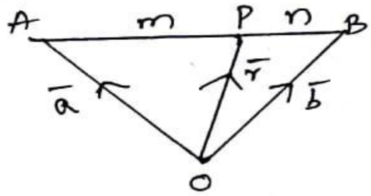
● Proof: Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$ and $\overline{OP} = \bar{r}$.

Let P divides AB in the ratio m:n internally



$$\frac{AP}{PB} = \frac{m}{n}$$

$$\frac{|\overline{AP}|}{|\overline{PB}|} = \frac{m}{n}.$$

(ie) $n|\overline{AP}| = m|\overline{PB}|$

But $\overline{AP} = \overline{OP} - \overline{OA}$
$= \bar{r} - \bar{a}$

$\overline{PB} = \overline{OB} - \overline{OP} = \bar{b} - \bar{r}$

$$\therefore n(\bar{r} - \bar{a}) = m(\bar{b} - \bar{r})$$

$$n\bar{r} - n\bar{a} = m\bar{b} - m\bar{r}$$

$$(n+m)\bar{r} = m\bar{b} + n\bar{a}$$

$$\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}$$

Note: 1) If P divides AB in the ratio m:n externally then

$$\bar{r} = \frac{m\bar{b} - n\bar{a}}{m-n}.$$

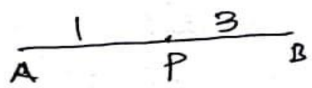
2) If P is the mid point of AB $m:n = 1:1$ then $\overline{OP}$

$$\bar{r} = \frac{\bar{a} + \bar{b}}{2}$$

Ex 8.3 Let A and B be two points with P.V. $2\bar{a} + 4\bar{b}$, $2\bar{a} - 8\bar{b}$. Find the P.V of the points which divides the line segment joining A and B in the ratio 1:3 internally and externally.

Internally $\overline{OA} = 2\bar{a} + 4\bar{b}$

$\overline{OB} = 2\bar{a} - 8\bar{b}$



ratio 1:3 internally.

$$\overline{OP} = \frac{1\overline{OB} + 3\overline{OA}}{1+3} = \frac{(2\bar{a} - 8\bar{b}) + 3(2\bar{a} + 4\bar{b})}{1+3}$$

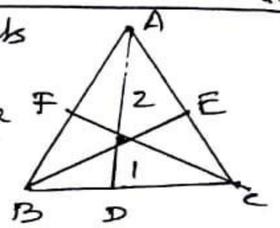
$$= \frac{8\bar{a} + 4\bar{b}}{4} = \frac{\cancel{4}(2\bar{a} + \bar{b})}{\cancel{4}}$$

Externally: $\overline{OP} = \frac{1\cdot\overline{OB} - 3\cdot\overline{OA}}{1-3} = \frac{(2\bar{a} - 8\bar{b}) - 3(2\bar{a} + 4\bar{b})}{-2}$

$$= \frac{-4\bar{a} - 20\bar{b}}{-2} = \frac{\cancel{-2}(2\bar{a} + 10\bar{b})}{\cancel{-2}}$$

Theorem 8.3) The medians of a $\triangle$le are concurrent

In a $\triangle$le ABC D, E, F are the mid points of BC, CA, AB. $\therefore$ AD, BE, CF are the medians.



Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$, $\overline{OC} = \bar{c}$

$\therefore \overline{OD} = \dfrac{\bar{b}+\bar{c}}{2}$, $\overline{OE} = \dfrac{\bar{c}+\bar{a}}{2}$ $\overline{OF} = \dfrac{\bar{a}+\bar{b}}{2}$

Let $G_1$ be the point on AD which divides AD in the ratio 2:1

$$\therefore \overline{OG_1} = \frac{2\cdot\overline{OD} + 1\cdot\overline{OA}}{2+1} = \frac{\cancel{2}\left(\frac{\bar{b}+\bar{c}}{\cancel{2}}\right)+1\cdot\bar{a}}{3} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$$

Let $G_2$ be the point on BE which divides in the ratio 2:1.

and $G_3$ be the point on CF which divides in the ratio 2:1

|||ly we can find that $\overline{OG_2} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{2}$

$\overline{OG_3} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{2}$

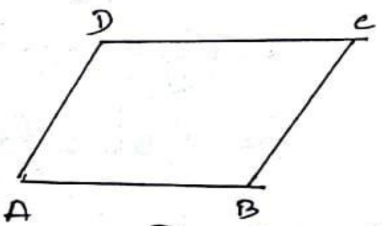
$\therefore G_1, G_2, G_3$ are the same point, Hence medians of a $\triangle$le are concurrent.

G is the centroid of the $\triangle$le. $\therefore \overline{OG} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

Theorem 8.4) A quadrilateral is a parallelogram iff its diagonals bisect each other.

Proof: Let ABCD be a quadrilateral with diagonals AC and BD.

$\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$, $\overline{OC} = \bar{c}$ $\overline{OD} = \bar{d}$



I If ABCD is a parallelogram

$\overline{AB} = \overline{DC}$

$\overline{OB} - \overline{OA} = \overline{OC} - \overline{OD}$

$\bar{b} - \bar{a} = \bar{c} - \bar{d}$

$\bar{b} + \bar{d} = \bar{a} + \bar{c}$

$\dfrac{\bar{a}+\bar{c}}{2} = \dfrac{\bar{b}+\bar{d}}{2}$

$\Rightarrow$ Mid points of AC and BD are same.

II If $\dfrac{\bar{a}+\bar{c}}{2} = \dfrac{\bar{b}+\bar{d}}{2} \Rightarrow \bar{a}+\bar{c} = \bar{b}+\bar{d}$

$\bar{c} - \bar{d} = \bar{b} - \bar{a}$

$\Rightarrow \overline{OC} - \overline{OD} = \overline{OB} - \overline{OA}$

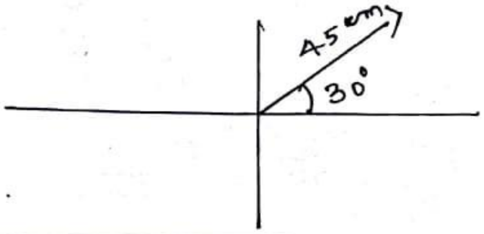
$\Rightarrow \overline{DC} = \overline{AB}$

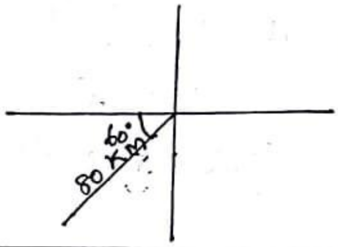
This shows that AB, CD are equal and parallel |||ly we can prove BC, AD are parallel and equal

$\therefore$ ABCD is parallelogram.

## EXERCISE – 8.1

1) Represent graphically the displacement of
1) 45cm 30° north of east 2) 80 KM, 60° south of west.




3) Let $\bar{a}$ and $\bar{b}$ be the po.v of the points A and B. P.T the P.V of the points which trisects the line segment AB are $\frac{\bar{a}+2\bar{b}}{3}$, $\frac{\bar{b}+2\bar{a}}{3}$.

Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$


Let P be a point which divides AB in the ratio 1 : 2 internally.

$$\overline{OP} = \frac{1\cdot\bar{b} + 2\cdot\bar{a}}{1+2} = \frac{\bar{b}+2\bar{a}}{3}.$$

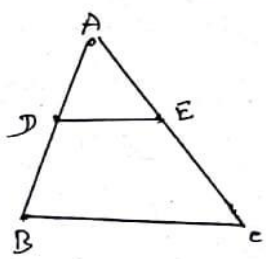
Let Q be the point on AB which divides AB in the ratio 2 : 1

$$\frac{2\cdot\bar{b} + \bar{a}}{2+1} = \frac{2\bar{b}+\bar{a}}{3}.$$

4) If D, E are the mid points of the sides AB and AC of a $\triangle$le ABC
P.T $\overline{BE}+\overline{DE} = \frac{3}{2}\overline{BC}$

$$\overline{BE} = \overline{BC}+\overline{CE}$$
$$= \overline{BC}+\tfrac{1}{2}\overline{CA}$$

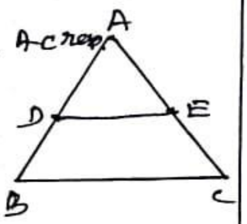
$$\overline{DC} = \overline{DB}+\overline{BC}$$
$$= \tfrac{1}{2}\overline{AB}+\overline{BC}$$




$$\therefore \overline{BE}+\overline{DC} = \overline{BC}+\tfrac{1}{2}\overline{CA}+\tfrac{1}{2}\overline{AB}+\overline{BC}$$
$$= 2\overline{BC}+\tfrac{1}{2}(\overline{CA}+\overline{AB})$$
$$= 2\overline{BC}+\tfrac{1}{2}(\overline{CB})$$
$$= 2\overline{BC}-\tfrac{1}{2}\overline{BC}$$
$$= 3/2\,\overline{BC}.$$

5) P.T the line segment joining the mid points of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.

In a $\triangle^{le}$ ABC D, E are the mid point of AB, AC resp.




$\therefore \overline{OD} = \dfrac{\overline{OA}+\overline{OB}}{2} \qquad \overline{OE} = \dfrac{\overline{OA}+\overline{OC}}{2}$

$$\overline{DE} = \overline{OE} - \overline{OD} = \frac{\overline{OA}+\overline{OC}}{2} - \frac{\overline{OA}+\overline{OB}}{2}$$

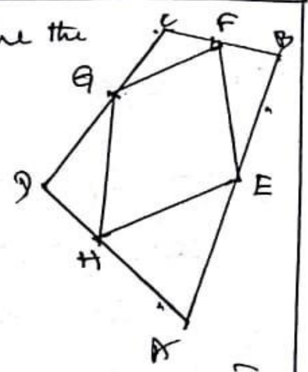
$$= \frac{\overline{OC}-\overline{OB}}{2} = \frac{1}{2}(\overline{BC})$$

$$|\overline{DE}| = \frac{1}{2}|\overline{BC}|$$

which implies that line joining of the mid points of the two sides is parallel to third side and length is equal to $\frac{1}{2}$ of the length of the third side.

6) P.T the line segments joining the mid points of the adjacent sides of a quadrilateral from a parallelogram.

ABCD is a quadrilateral in which E, F, G, H are the are the mid points of AB, BC, CD, DA resp.




$\overline{OE} = \dfrac{\overline{OA}+\overline{OB}}{2} \qquad \overline{OG} = \dfrac{\overline{OD}+\overline{OC}}{2}$

$\overline{OF} = \dfrac{\overline{OB}+\overline{OC}}{2} \qquad \overline{OH} = \dfrac{\overline{OD}+\overline{OA}}{2}$

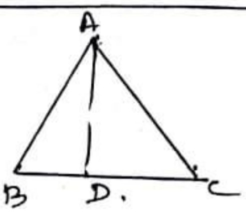
$$\text{Mid point of } EG = \frac{\frac{\overline{OA}+\overline{OB}}{2} + \frac{\overline{OC}+\overline{OD}}{2}}{2} = \frac{\overline{OA}+\overline{OB}+\overline{OC}+\overline{OD}}{4}.$$

$$\text{Mid point of } FH = \frac{\frac{\overline{OB}+\overline{OC}}{2} + \frac{\overline{OD}+\overline{OA}}{2}}{2} = \frac{\overline{OA}+\overline{OB}+\overline{OC}+\overline{OD}}{4}.$$

$\therefore$ The mid points of the diagonals EG and FH are same EFGH is a parallelogram.

9) If D is the mid point of the side BC of a $\triangle$ ABC P.T $\overline{AB}+\overline{AC}=2\overline{AD}$.

Let D be the mid point of BC of the $\triangle$ ABC




$$\overline{AB} = \overline{AD}+\overline{DB} \quad \text{——— (1)}$$

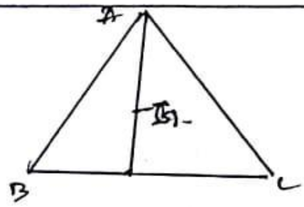
$$\overline{AC} = \overline{AD}+\overline{DC}$$

$$= \overline{AD}-\overline{DB} \text{ — (2)} \quad \because \text{ D is the midepoint of BC and DC and DB are opposite}$$

$$\overline{AB}+\overline{AC} = 2\overline{AD}$$

10) If G is the centroid of the $\triangle$le ABC P.T $\overline{GA}+\overline{GB}+\overline{GC}=\overline{0}$

Let G be the centroid of the $\triangle$le ABC




$$\overline{OG} = \frac{\overline{OA}+\overline{OB}+\overline{OC}}{3.}$$

$$3\overline{OG} = \overline{OA}+\overline{OB}+\overline{OC} \quad \text{——— (*)}$$

Given $\overline{GA}+\overline{GB}+\overline{GC} = \overline{OA}-\overline{OG}+\overline{OB}-\overline{OG}+\overline{OC}-\overline{OG}$

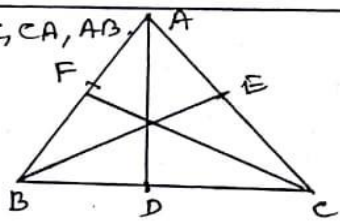
$$= (\overline{OA}+\overline{OB}+\overline{OC}) - 3\overline{OG}$$

$$= 3\overline{OG} - 3\overline{OG}$$

$$= \overline{0}$$

11) Let A, B, C are the vertices of a $\triangle$le. Let D, E and F are the mid points of the sides BC, CA, AB resp. S.T $\overline{AD}+\overline{BE}+\overline{CF}=0$.

Let D, E, F are the mid points of BC, CA, AB




$$\overline{AD} = \overline{AB}+\overline{BD} = \overline{AB}+\tfrac{1}{2}\overline{BC}$$

$$\overline{BE} = \overline{BC}+\overline{CE} = \overline{BC}+\tfrac{1}{2}\overline{CA}$$

$$\overline{CF} = \overline{CA}+\overline{AF} = \overline{CA}+\tfrac{1}{2}\overline{AB}$$

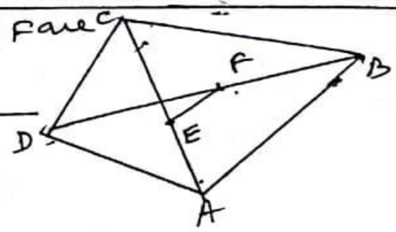
$$\overline{AD}+\overline{BE}+\overline{CF} = \tfrac{3}{2}\overline{AB}+\tfrac{3}{2}\overline{BC}+\tfrac{3}{2}\overline{CA}$$

$$= \tfrac{3}{2}(\overline{AB}+\overline{BC}+\overline{CA})$$

$$= 3/2(0) = 0$$

12) If ABCD is a quadrilateral and E, F are the mid points of AC and BD resp then P.T $\overline{AB}+\overline{AD}+\overline{CB}+\overline{CD}=4\overline{EF}$

ABCD is a quadrilateral in which, E, F are the mid points of AC, BD




$\overline{AB} = \overline{AE} + \overline{EF} + \overline{FB}$

$\overline{AD} = \overline{AE} + \overline{EF} + \overline{FD}$

$\overline{CB} = \overline{CE} + \overline{EF} + \overline{FB}$

$\overline{CD} = \overline{CE} + \overline{EF} + \overline{FD}$

$\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD} = 2(\overline{AE} + \overline{CE}) + 4\overline{EF} + 2(\cancel{\overline{FB}}\ \overline{FB} + \overline{FD})$

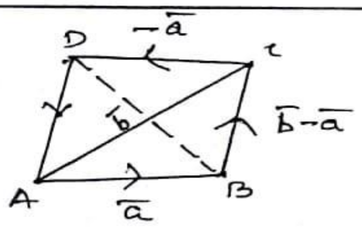
$= 2(0) + 4\overline{EF} + 2(0)$

$= 4\overline{EF}$

$\because$ $\overline{AE}, \overline{CE}$ are equal but opposite
$\overline{FB}, \overline{FD}$ are equal but opposite.

7) If $\bar{a}, \bar{b}$ represent a side and a diagonal of a parallelogram find the other sides and the other diagonal.

In a parallelogram ABCD




$\overline{AB} = \bar{a}, \quad \overline{AC} = \bar{b}$

By T.L.A.

$\overline{AB} + \overline{BC} = \overline{AC}$

$\overline{BC} = \overline{AC} - \overline{AB}$

$= \bar{b} - \bar{a}$

$\overline{DA}$ is equal and opposite to $\overline{BC}$

$\therefore \overline{DA} = -(\bar{a} - \bar{b})$

$\overline{CD}$ is equal and opposite to $\overline{AB}$

$\overline{CD} = -\bar{a}$.

$\overline{BD} = \overline{BC} + \overline{CD}$

$= (\bar{b} - \bar{a}) + (-\bar{a}) = \bar{b} - 2\bar{a}$

8) If $\overline{PO} + \overline{OQ} = \overline{QO} + \overline{OR}$ P.T the points P, Q, R are collinear.

Given $\overline{PO} + \overline{OQ} = \overline{QO} + \overline{OR}$

$\overline{PQ} = \overline{QR}$ —— (1).


Again $\overline{PQ} + \overline{QR} = \cancel{\overline{OQ}} - \overline{OP} + \overline{OR} - \cancel{\overline{OQ}}$

$= \overline{OR} - \overline{OP}$

$= \overline{PR}$ —— (2)

From (1) and (2). P, Q, R are collinear.

EXERCISE – 8.2.

Ex 8.4. Find the unit vector along the direction of $5\hat{i} - 3\hat{j} + 4\hat{k}$

Let $\bar{a} = 5\hat{i} - 3\hat{j} + 4\hat{k}$

$|\bar{a}| = \sqrt{25+9+16} = \sqrt{50}$

$\hat{a} = \frac{\bar{a}}{|\bar{a}|} = \frac{5\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{50}}$.

Note! unit vector parallel to $\bar{a}$ but in the opposite direction

$\hat{a} = -\frac{\bar{a}}{|\bar{a}|}$.

Ex: 8.5) Find the direction ratios and direction cosines of

i) $3\hat{i} + 4\hat{j} - 6\hat{k}$ 2) $3\hat{i} - 4\hat{k}$

Let $\bar{a} = 3\hat{i} + 4\hat{j} - 6\hat{k}$

Dr's of $\bar{a}$ = 3, 4, −6. $|\bar{a}| = \sqrt{9+16+36} = \sqrt{61}$

Dc's $\left(\frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{-6}{\sqrt{61}}\right)$

Let $\bar{b} = 3\hat{i} - 4\hat{k}$

Dr's of $\bar{b}$ 3, 0, −4. $|\bar{b}| = \sqrt{9+16} = 5$

DCs $\left(\frac{3}{5}, 0, -\frac{4}{5}\right)$

Ex 8.6) 1) Find the direction cosines of a vector whose dr's are 2, 3, −6.

2) Can a vector have direction angles $30^\circ, 45^\circ, 60^\circ$

3) Find the dc's of $\overline{AB}$ where A (2, 3, 1) B (3, −1, 2)

4) Find the dc's of the line joining (2, 3, 1) (3, −1, 2)

5) The dr's of a vector are 2, 3, 6 and its magnitude is 5. Find the vector.

1) dc's are $\frac{2}{\sqrt{4+9+36}}, \frac{3}{\sqrt{49}}, \frac{-6}{\sqrt{49}}$

(ie) $\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right)$

2) If $\alpha, \beta, \gamma$ are the angles with OX, OY, OZ then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(ie) $\cos^2 30 + \cos^2 45 + \cos^2 60 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \neq 1$.

∴ They are not the direction angles of a vector.

3) $\overline{AB} = \overline{OB} - \overline{OA} = \hat{i} - 4\hat{j} + \bar{k}$

$|\overline{AB}| = \sqrt{1+16+1} = \sqrt{18}$.

$\therefore$ DC's $\left(\frac{1}{\sqrt{18}}, \frac{-4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right)$

---

4) $\overline{AB} = \overline{OB} - \overline{OA} = \hat{i} - 4\hat{j} + \bar{k}$.

The dc's of AB are $\left(\frac{1}{\sqrt{18}}, \frac{-4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right)$

Suppose if we take second point as first point

dc's are $\left(\frac{-1}{\sqrt{18}}, \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right)$

---

5) dc's are $\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

$\therefore$ unit vector is $\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\bar{k}$

The required vector is $\frac{5}{7}(2\hat{i} + 3\hat{j} + 6\bar{k})$

---

EX 8.7) S.T the points whose PV are $2\hat{i}+3\hat{j}-5\bar{k}$, $3\hat{i}+\hat{j}-2\bar{k}$ and $6\hat{i}-5\hat{j}+7\bar{k}$ are collinear.

Let $\overline{OA} = 2\hat{i}+3\hat{j}-5\bar{k}$

$\overline{OB} = 3\hat{i}+\hat{j}-2\bar{k}$

$\overline{OC} = 6\hat{i}-5\hat{j}+7\bar{k}$.

$\overline{AB} = \overline{OB} - \overline{OA} = \hat{i} - 2\hat{j} + 3\bar{k}$

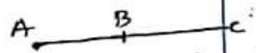
$\overline{AC} = \overline{OC} - \overline{OA} = 4\hat{i} - 8\hat{j} + 12\bar{k}$

$= 4(\hat{i} - 2\hat{j} + 3\bar{k})$

$\overline{AC} = 4\,\overline{AB}$

$\therefore$ The three Points are collinear.

Note: If $\overline{AC} = 4\overline{AB}$, $\overline{AC}$ and $\overline{AB}$ are ||el




when the three points are collinear then only AB and AC are ||el.

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EX 8.8) Find the point whose P.V has magnitude 5 and ||el to $4\hat{i} - 3\hat{j} + 10\bar{k}$

Let $\bar{a} = 4\hat{i} - 3\hat{j} + 10\bar{k}$

$|\bar{a}| = \sqrt{16+9+100} = \sqrt{125} = 5\sqrt{5}$

$\hat{a} = \frac{4\hat{i} - 3\hat{j} + 10\bar{k}}{5\sqrt{5}}$

$5\hat{a} = \frac{\cancel{5}(4\hat{i} - 3\hat{j} + 10\bar{k})}{\cancel{5}\sqrt{5}}$ $\therefore$ Required points are $\left(\frac{4}{\sqrt{5}}, \frac{-3}{\sqrt{5}}, \frac{10}{\sqrt{5}}\right)$ $\frac{10}{\sqrt{5}} = 2\sqrt{5}$

EX 8.9) P.T the points whose pv's $2\hat{i}+4\hat{j}+3\hat{k}$, $4\hat{i}+\hat{j}+9\hat{k}$, $10\hat{i}-\hat{j}+6\hat{k}$ forms a rt triangle

let $\bar{OA} = 2\hat{i}+4\hat{j}+3\hat{k}$ $\quad \bar{AB} = \bar{OB}-\bar{OA} = 2\hat{i}-3\hat{j}+6\hat{k}$

$\bar{OB} = 4\hat{i}+\hat{j}+9\hat{k}$ $\quad \bar{BC} = \bar{OC}-\bar{OB} = 6\hat{i}-2\hat{j}-3\hat{k}$

$\bar{OC} = 10\hat{i}-\hat{j}+6k$ $\quad \bar{CA} = \bar{OA}-\bar{OC} = -8\hat{i}+5\hat{j}-3\hat{k}$

$\bar{AB}+\bar{BC}+\bar{CA} = 0$ ∴ First it is a triangle.

$|\bar{AB}| = \sqrt{4+9+36} = 7$

$|\bar{BC}| = \sqrt{36+4+9} = 7$

$|\bar{CA}| = \sqrt{64+25+9} = \sqrt{98}$.

$|AB|^2 + |\bar{BC}|^2 = |\bar{CA}|^2$

$49+49 = 98$. ∴ given points forms a rt $\triangle$le.

EX 8.10) S.T the vectors $5\hat{i}+6\hat{j}+7\hat{k}$, $7\hat{i}-8\hat{j}+9\hat{k}$, $3\hat{i}+20\hat{j}+5\hat{k}$ are coplanar.

Let $5\hat{i}+6\hat{j}+7\hat{k} = s(7\hat{i}-8\hat{j}+9\hat{k})+t(3\hat{i}+20\hat{j}+5\hat{k})$

$7s+3t = 5$ — ①

$-8s+20t = 6$ — ②

$9s+5t = 7$ — ③

① × 5 $\quad 35s+15t = 25$

③ × 3 $\quad 27s+15t = 21$

$8s = 4$

$s = 1/2$

$t = 1/2$

∴ $5\hat{i}+6\hat{j}+7\hat{k} = \frac{1}{2}(7\hat{i}-8\hat{j}+9\hat{k})+1/2(3\hat{i}+20\hat{j}+5\hat{k})$

∴ we can write one vector is a linear combination of other two vectors. Hence given vectors are coplanar.

7) 8.2) S.T the vectors $2\hat{i}-\hat{j}+\hat{k}$, $3\hat{i}-4\hat{j}-4\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ form a right angled $\triangle$le.

$AB = 2\hat{i}-\hat{j}+\hat{k}$ $\quad |\bar{AB}| = \sqrt{4+1+1} = \sqrt{6}$

$\bar{BC} = 3\hat{i}-4\hat{j}-4\hat{k}$ $\quad |\bar{BC}| = \sqrt{9+16+16} = \sqrt{41}$

$\bar{CA} = \hat{i}-3\hat{j}-5\hat{k}$ $\quad |\bar{CA}| = \sqrt{1+9+25} = \sqrt{35}$

$\bar{CA}+\bar{AB} = \bar{BC}$ ∴ First it is a $\triangle$le.

$|\bar{CA}|^2+|\bar{AB}|^2 = |\bar{BC}|^2$

$35+6 = 41$. ∴ it is rt angled $\triangle$le.

8) 8.2) Find the value of $\lambda$ for which the vectors $\bar{a} = 3\hat{i}+2\hat{j}+9\hat{k}$ $\bar{b} = \hat{i}+\lambda\hat{j}+3\hat{k}$ are parallel.

If $\bar{a}$ and $\bar{b}$ are $\parallel$el $\bar{b} = t\bar{a}$

$$\hat{i}+\lambda\hat{j}+3\hat{k} = 3\left(\hat{i}+\frac{2}{3}\hat{j}+3\hat{k}\right)$$
$$\Rightarrow \lambda = 2/3.$$

9) 8.2 S.T the following vectors are coplanar.

1) $\hat{i}-2\hat{j}+3\hat{k}$ $-2\hat{i}+3\hat{j}-4\hat{k}$, $-\hat{j}+2\hat{k}$

2) $5\hat{i}+6\hat{j}+7\hat{k}$, $7\hat{i}-8\hat{j}+9\hat{k}$, $3\hat{i}+20\hat{j}+5\hat{k}$.

1) $\hat{i}-2\hat{j}+3\hat{k} = s(-2\hat{i}+3\hat{j}-4\hat{k})+t(-\hat{j}+2\hat{k})$

$$1 = -2s \Rightarrow s = -1/2$$
$$-2 = 3s-t \Rightarrow -2 = 3(-1/2)-t$$
$$t = -\frac{3}{2}+2 = 1/2.$$

$\therefore \hat{i}-2\hat{j}+3\hat{k} = -\frac{1}{2}(-2\hat{i}+3\hat{j}-4\hat{k})+\frac{1}{2}(-\hat{j}+2\hat{k})$

$\therefore$ we can write one vector as a linear combination of other two vectors. Hence they are coplanar.

2) $5\hat{i}+6\hat{j}+7\hat{k} = s(7\hat{i}-8\hat{j}+9\hat{k})+t(3\hat{i}+20\hat{j}+5\hat{k})$

$5 = 7s+3t$ — ①

$6 = -8s+20t$ — ② This is Example 8.10.

$7 = 9s+5t$ — ③

10) 8.2) S.T the points whose P.V $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

$\overline{OA} = 4\hat{i}+5\hat{j}+\hat{k}$

$\overline{OB} = -\hat{j}-\hat{k}$

$\overline{OC} = 3\hat{i}+9\hat{j}+4\hat{k}$

$\overline{OD} = -4\hat{i}+4\hat{j}+4\hat{k}$

$\overline{AB} = -4\hat{i}-6\hat{j}-2\hat{k}$ $(\overline{OB}-\overline{OA})$

$\overline{BC} = 3\hat{i}-6\hat{j}-2\hat{k}$

$\overline{CD} = 7\hat{i}-5\hat{j}$.

$-4\hat{i}-6\hat{j}-2\hat{k} = s(3\hat{i}-6\hat{j}-2\hat{k})+t(7\hat{i}-5\hat{j})$

$$-4 = 3s+7t$$
$$-6 = 10s-5t$$
$$-2 = 5s \Rightarrow s = -2/5$$

$-4 = 3(-2/5) - 7t$

$7t = -\frac{6}{5} + 4 = \frac{-6+20}{5}$

$= \frac{14}{5} \Rightarrow t = \frac{14}{5 \times 7} 2$

$\therefore -4i - 6j - 2k = -\frac{2}{5}(3i - 6j - 2k) + \frac{2}{5}(-7i - 5j)$

$\therefore$ one vector can be written as sum of two linear vectors. Hence the given vectors are coplanar.

11/8.2) If $\bar{a} = 2i + 3j - 4\bar{k}$, $\bar{b} = 3\bar{i} - 4j - 5\bar{k}$, $\bar{c} = -3\hat{i} + 2\hat{j} + 3\bar{k}$. Find the magnitude d.c's of 1) $\bar{a} + \bar{b} + \bar{c}$ 2) $3\bar{a} - 2\bar{b} + 5\bar{c}$.

$\bar{a} = 2\hat{i} + 3\hat{j} - 4\bar{k}$

$\bar{b} = 3\hat{i} - 4\hat{j} - 5\bar{k}$

$\bar{c} = -3\hat{i} + 2\hat{j} + 3\bar{k}$.

(1) $\bar{a} + \bar{b} + \bar{c} = 2\hat{i} + 1\hat{j} - 6\bar{k}$

$|\bar{a} + \bar{b} + \bar{c}| = \sqrt{4 + 1 + 36} = \sqrt{41}$

d.c's : $\frac{2}{\sqrt{41}}, \frac{1}{\sqrt{41}}, \frac{-6}{\sqrt{41}}$

$3\bar{a} = 6\bar{i} + 9\hat{j} - 12\bar{k}$

$-2\bar{b} = -6\bar{i} + 8\hat{j} + 10\bar{k}$.

$5\bar{c} = -15\bar{i} + 10\hat{j} + 15\bar{k}$.

$3\bar{a} - 2\bar{b} + 5\bar{c} = -15\bar{i} + 27\hat{j} + 13\bar{k}$

$|3\bar{a} - 2\bar{b} + 5\bar{c}| = \sqrt{1123}$

d.c's : $\frac{-15}{\sqrt{1123}}, \frac{27}{\sqrt{1123}}, \frac{13}{\sqrt{1123}}$.

12/8.2) The P.V of the vertices of a triangle are $\hat{i} + 2\hat{j} + 3\bar{k}$, $3\hat{i} - 4\hat{j} + 5\bar{k}$ and $-2\bar{i} + 3\hat{j} - 7\bar{k}$. Find the perimeter of the $\triangle$le –

$\vec{OA} = \hat{i} + 2\hat{j} + 3\bar{k}$

$\vec{OB} = 3\bar{i} - 4\hat{j} + 5\bar{k}$

$\vec{OC} = -2\hat{i} + 3\hat{j} - 7\bar{k}$.

$\vec{AB} = \vec{OB} - \vec{OA} = 2\hat{i} - 6\hat{j} + 2\bar{k}$

$\vec{BC} = \vec{OC} - \vec{OB} = -5\hat{i} + 7\hat{j} - 12\bar{k}$

$\vec{CA} = \vec{OA} - \vec{OC} = 3\hat{i} - \hat{j} + 10\bar{k}$.

$|\vec{AB}| = \sqrt{4 + 36 + 4} = \sqrt{44}$

$|\vec{BC}| = \sqrt{25 + 49 + 144} = \sqrt{218}$

$|\vec{CA}| = \sqrt{9 + 1 + 100} = \sqrt{110}$

$\therefore$ Perimeter of the $\triangle$le $= \sqrt{44} + \sqrt{218} + \sqrt{110}$.

$\frac{13}{8.2}$) Find the unit vector parallel to $3\bar{a}-2\bar{b}+4\bar{c}$ if $\bar{a}=3\bar{i}-\bar{j}-4\bar{k}$ $\bar{b}=-2\bar{i}+4\bar{j}-3\bar{k}$, $\bar{c}=\bar{i}+2\bar{j}-\bar{k}$

$3\bar{a}=9\bar{i}-3\bar{j}-12\bar{k}$

$-2\bar{b}=4\bar{i}-8\bar{j}+6\bar{k}$

$4\bar{c}=4\bar{i}+8\bar{j}-4\bar{k}$

$3\bar{a}-2\bar{b}+4\bar{c}$
$=17\bar{i}+3\bar{j}+10\bar{k}$

$|17\bar{i}+3\bar{j}+10\bar{k}|=\sqrt{289+9+100}$
$=\sqrt{398}$

unit vector $\parallel^{el}$ to $3\bar{a}-2b+4\bar{c}$ $=\dfrac{17\bar{i}+3\bar{j}+10\bar{k}}{\sqrt{398}}$

$\frac{15}{8.2}$) The PV of ~~a, b, c, d are~~ the points P, Q, R, S are $\bar{i}+\bar{j}+\bar{k}$, $2\bar{i}+5\bar{j}$, $3\bar{i}+2\bar{j}-3\bar{k}$ and $\bar{i}-6\bar{j}-\bar{k}$ resp.
P.T the lines PQ and RS are parallel.

$\bar{OP}=\bar{i}+\bar{j}+\bar{k}$

$\bar{OQ}=2\bar{i}+5\bar{j}$

$\bar{OR}=3\bar{i}+2\bar{j}-3\bar{k}$

$\bar{OS}=\bar{i}-6\bar{j}-\bar{k}$

$\bar{PQ}=\bar{OQ}-\bar{OP}=\bar{i}+4\bar{j}-\bar{k}$

$\bar{RS}=\bar{OS}-\bar{OR}=-2\bar{i}-8\bar{j}+2\bar{k}$

$\bar{RS}=-2(\bar{i}+4\bar{j}-\bar{k})$
$=-2\bar{PQ}$

$\therefore$ PQ, RS are parallel.

$\frac{16}{8.2}$) Find the value or values of m for which $m(\bar{i}+\bar{j}+\bar{k})$ is a unit vector

$\hat{a}=\dfrac{\bar{a}}{|\bar{a}|}=\pm m(\bar{i}+\bar{j}+\bar{k})$
$=\pm\dfrac{(\bar{i}+\bar{j}+\bar{k})}{1/m}$

$\Rightarrow |\bar{a}|=\sqrt{1+1+1}=\sqrt{3}$,
$|\bar{a}|=\pm\dfrac{1}{m}$
$m=\pm\dfrac{1}{\sqrt{3}}$.

$\frac{17}{8.2}$) S.T the points A (1,1,1) B (1,2,3) C (2,−1,1) are the vertices of isosceles triangle

$\bar{OA}=\bar{i}+\bar{j}+\bar{k}$

$\bar{OB}=\bar{i}+2\bar{j}+3\bar{k}$

$\bar{OC}=2\bar{i}-\bar{j}+\bar{k}$.

$\bar{AB}=\bar{j}+2\bar{k}$ $\quad |\bar{AB}|=\sqrt{1+4}=\sqrt{5}$

$\bar{BC}=\bar{i}-3\bar{j}-2\bar{k}$ $\quad |\bar{BC}|=\sqrt{1+9+4}=\sqrt{14}$

$\bar{CA}=-\bar{i}+2\bar{j}$ $\quad |\bar{CA}|=\sqrt{1+4}=\sqrt{5}$

$|\bar{AB}|=|\bar{CA}|$ $\therefore$ it is isosceles $\triangle$le.

$\frac{1}{8.2}$) Verify whether the following ratios are dc's of some vector or not.

i) $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}$ 2) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$ 3) $\frac{4}{3}, 0, \frac{3}{4}$.

W.K.T $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

1) $\frac{1}{25} + \frac{9}{25} + \frac{16}{25} \neq 1$ ∴ not dc's

2) $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$ ∴ 2 is dc's

3) $\frac{16}{9} + 0 + \frac{9}{16} \neq 1$. ∴ not dc's.

$\frac{2}{8.2}$) Find the direction cosines of a vector whose dr's are

1) 1, 2, 3 2) 3, −1, 3 3) 0, 0, 7.

1) Let $\bar{r} = i + 2j + 3\bar{k}$

$|\bar{r}| = \sqrt{1+4+9} = \sqrt{14}$ ∴ dc's $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.

2) Let $\bar{r} = 3i - j + 3\bar{k}$

$|\bar{r}| = \sqrt{9+1+9} = \sqrt{19}$ dc's $\left(\frac{3}{\sqrt{19}}, \frac{-1}{\sqrt{19}}, \frac{3}{\sqrt{19}}\right)$

3) Let $\bar{r} = 7\bar{k}$

$|\bar{r}| = \sqrt{49} = 7$ dc's $\left(0, 0, \frac{7}{7}\right)$

$\frac{3}{8.2}$) Find the dc's and dr's of the following vectors.

1) $3\bar{i} - 4j + 8\bar{k}$ 2) $3i + j + \bar{k}$ 3) $\hat{j}$ 4) $5\bar{i} - 3\bar{j} - 48\bar{k}$

5) $3i - 3\bar{k} + 4\bar{j}$ 6) $i - \bar{k}$.

| | |
|---|---|
| Let 1) $\bar{x} = 3i - 4j + 8\bar{k}$ <br> $\lvert\bar{x}\rvert = \sqrt{9+16+64} = \sqrt{89}$ | dr's : (3, −4, 8) <br> dc's $\left(\frac{3}{\sqrt{89}}, -\frac{4}{\sqrt{89}}, \frac{8}{\sqrt{89}}\right)$ |
| 2) $\bar{x} = 3i + j + k$ <br> $\lvert\bar{x}\rvert = \sqrt{9+1+1} = \sqrt{10}$ | dr's (3, 1, 1) <br> dc's $\left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$ |
| 3 $\bar{x} = \bar{j}$ <br> $\lvert\bar{x}\rvert = \sqrt{1} = 1$ | dr's (0, 1, 0) <br> dc's (0, 1, 0) |

| | |
|---|---|
| 4) $\bar{x} = 5\hat{i} - 3\hat{j} + 48\hat{k}$<br>$\lvert\bar{x}\rvert = \sqrt{2338}$ | d r's $(5, -3, -48)$<br>dc's $\left(\frac{5}{\sqrt{2338}}, \frac{-3}{\sqrt{2338}}, \frac{-48}{\sqrt{2338}}\right)$. |
| 5) $\bar{x} = 3\hat{i} + 4\hat{j} - 3\hat{k}$<br>$\lvert\bar{x}\rvert = \sqrt{9+16+9} = \sqrt{34}$ | d r's $(3, 4, -3)$<br>dcs' $\left(\frac{3}{\sqrt{34}}, \frac{4}{\sqrt{34}}, \frac{-3}{\sqrt{34}}\right)$ |
| 6) $\bar{x} = \hat{i} - \hat{k}$<br>$\lvert\bar{x}\rvert = \sqrt{1+1} = \sqrt{2}$ | d r's $(1, 0, -1)$<br>dcs $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$ |

$\frac{5)}{8.2}$ If $\frac{1}{2}, \frac{1}{\sqrt{2}}$, a are the direction cosines of some vector find a.

$\cos\alpha = \frac{1}{2}$, $\cos\beta = \frac{1}{\sqrt{2}}$, $\cos\gamma = a$

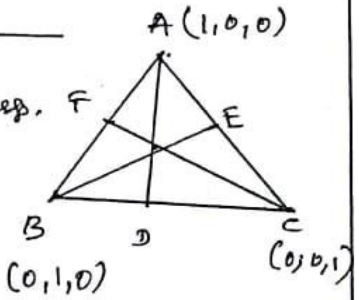
W.K.T $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\frac{1}{4} + \frac{1}{2} + a^2 = 1$$

$$a^2 = \frac{1}{4}$$

$$a = \pm \frac{1}{2}$$

$\frac{4}{8.2})$ A triangle is formed by joining the points $(1,0,0)$ $(0,1,0)$ $(0,0,1)$. Find the dc's of the medians.




A $(1,0,0)$ B $(0,1,0)$ C $(0,0,1)$

Let D, E, F are the mid points of BC, CA, AB resp.

AD, BE, CF are the medians.

Mid point of BC is D $= \left(0, \frac{1}{2}, \frac{1}{2}\right)$

A $= (1, 0, 0)$

D.r's of AD $= \left(-1, \frac{1}{2}, \frac{1}{2}\right)$ $\quad \sqrt{1 + \frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{\cancel{6}3}{\cancel{4}2}}$

DCs of AD $= \left(-\frac{1 \cdot \sqrt{2}}{\sqrt{3}}, \frac{\frac{1}{2} \times \sqrt{2}}{\sqrt{3}}, \frac{\frac{1}{2} \times \sqrt{2}}{\sqrt{3}}\right)$

$$= \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

Illy the dc's of other medians are $\left(\frac{1}{\sqrt{6}}, \frac{-\sqrt{2}}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$, $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$