UNIT 4–WORK, ENERGY AND POWER

GREETINGS Students, This class we are going to discuss about Motion in a vertical circle,power.

Motion in a vertical circle

Imagine that a body of mass (m) attached to one end of a mass less and inextensible string executes circular motion in a vertical plane with the other end of the string fixed. The length of the string becomes the radius (ρ r) of the circular path.

Let us discuss the motion of the body by taking the free body diagram (FBD) at ρa position where the position vector (r) makes an angle θ with the vertically downward direction and the instantaneous velocity.

There are two forces acting on the mass.

1. Gravitational force which acts downward
2. Tension along the string.

Applying Newton's second law on the mass, In the tangential direction,

$$mg\sin\theta = m a_t$$
$$mg\sin\theta = -m\left(\frac{dv}{dt}\right)$$

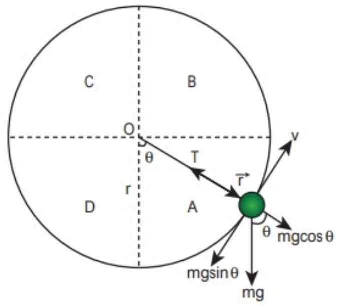

Figure 4.14 Motion in vertical circle

where, $a_t = -\frac{dv}{dt}$ is tangential retardation

In the radial direction,

$$T - mg\cos\theta = m a_r$$
$$T - mg\cos\theta = \frac{mv^2}{r} \qquad (4.29)$$

where, $a_r = \frac{v^2}{r}$ is the centripetal acceleration

The circle can be divided into four sections A, B, C, D for better understanding of themotion. The four important facts to be understood from the two equations are as follows:

(i) The mass is having tangential acceleration (g sin θ) for all values of θ (except θ = 0o), is clear that this vertical cirular motion is not a uniform circular motion.

(ii) From the equations (4.28) and (4.29) it is understood that as the magnitude of velocity is not a constant in the course of motion, the tension in the string is also not constant.

(iii) The equation (4.29), $T = mg\cos\theta + \frac{mv^2}{r}$ highlights that in sections A and D of the circle, $\left(\text{for } -\frac{\pi}{2} < \theta < \frac{\pi}{2};\ \cos\theta \text{ is positive}\right)$, the term mg cosθ is always greater than zero. Hence the tension cannot vanish even when the velocity vanishes.

(iv) The equation (4.29), $\frac{mv^2}{r} = T - mg\cos\theta$; further highlights that in sections B and C of the circle, $\left(\text{for } \frac{\pi}{2} < \theta < \frac{3\pi}{2};\ \cos\theta \text{ is negative}\right)$, the second term is always greater than zero. Hence velocity cannot vanish, even when the tension vanishes.

- These points are to be kept in mind while solving problems related to motion in vertical circle.
- To start with let us consider only two positions, say the lowest point 1 and the highest point 2 as shown in Figure 4.15 for further analysis.
- Let the velocity of the body ρat the lowest point 1 be v_1 , at the highest ρpoint 2 be v_2 ρand v at any other point. The direction of velocity is tangential to ρthe circular path at all points.
- Let T1 be the tension in the string at the lowest point and ρ T2 ρbe the tension at the highest point and T be the tension at any other point.
- Tension at each point acts towards the centre. The tensions and velocities at these two points can be found by applying the law of conservation of energy.

For the lowest point (1)

When the body is at the lowest point 1, the ρgravitational force m g which acts on the body (vertically downwards) and another ρ one is the tension T1 acting vertically upwards, i.e. towards the centre.

$$T_1 - mg = \frac{mv_1^2}{r} \quad (4.30)$$

$$T_1 = \frac{mv_1^2}{r} + mg \quad (4.31)$$

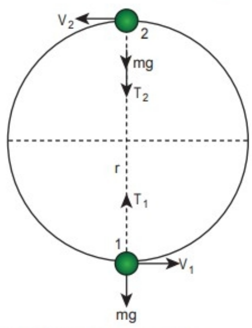

Figure 4.15 Motion in vertical circle shown for lowest and highest points

For the highest point (2)

At the highest point 2, both the gravitational ρforce mg ρon the body and the tension T2 act downwards, i.e. towards the centre again.

$$T_2 + mg = \frac{mv_2^2}{r} \quad (4.32)$$

$$T_2 = \frac{mv_2^2}{r} - mg \quad (4.33)$$

From equations (4.31) and (4.33), it is understood that $T_1 > T_2$. The difference in tension $T_1 - T_2$ is obtained by subtracting equation (4.33) from equation (4.31).

$$T_1 - T_2 = \frac{mv_1^2}{r} + mg - \left(\frac{mv_2^2}{r} - mg\right)$$

$$= \frac{mv_1^2}{r} + mg - \frac{mv_2^2}{r} + mg$$

$$T_1 - T_2 = \frac{m}{r}\left[v_1^2 - v_2^2\right] + 2mg \quad (4.34)$$

The term $\left[v_1^2 - v_2^2\right]$ can be found easily by applying law of conservation of energy at point 1 and also at point 2.

Total Energy at point 1(E_1) is same as the total energy at a point 2 (E_2)

$$E_1 = E_2 \quad (4.35)$$

Potential Energy at point 1, $U_1 = 0$ (by taking reference as point 1)

Kinetic Energy at point 1, $KE_1 = \frac{1}{2}mv_1^2$

Total Energy at point 1, $E_1 = U_1 + KE_1 =$

$= 0 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_1^2$

Similarly, Potential Energy at point 2, $U_2 = mg\,(2r)$ (h is 2r from point 1)

Kinetic Energy at point 2, $KE_2 = \frac{1}{2}mv_2^2$

Total Energy at point 2, $E_2 = U_2 + KE_2 =$

$2\ mg\,r + \frac{1}{2}mv_2^2$

From the law of conservation of energy given in equation (4.35), we get

$$\frac{1}{2}mv_1^2 = 2mgr + \frac{1}{2}mv_2^2$$

After rearranging,

$$\frac{1}{2}m\left(v_1^2 - v_2^2\right) = 2mgr$$

$$v_1^2 - v_2^2 = 4gr \quad (4.36)$$

Substituting equation (4.36) in equation (4.34) we get,

$$T_1 - T_2 = \frac{m}{r}\left[4gr\right] + 2mg$$

Therefore, the difference in tension is

$$T_1 - T_2 = 6\ mg \quad (4.37)$$

Minimum speed at the highest point (2) The body must have a minimum speed at point 2 otherwise, the string will slack before reaching point 2 and the body will not loop the circle. To find this minimum speed let us take the tension T2 = 0 in equation.

$$0 = \frac{mv_2^2}{r} - mg$$

$$\frac{mv_2^2}{r} = mg$$

$$v_2^2 = rg$$

$$v_2 = \sqrt{gr}$$

The body must have a speed at point 2, $v_2 \geq \sqrt{gr}$ to stay in the circular path.

Minimum speed at the lowest point 1

To have this minimum speed $\left(v_2 = \sqrt{gr}\right)$ at point 2, the body must have minimum speed also at point 1.

By making use of equation (4.36) we can find the minimum speed at point 1.

$$v_1^2 - v_2^2 = 4gr$$

Substituting equation (4.38) in (4.36),

$$v_1^2 - gr = 4gr$$

$$v_1^2 = 5gr$$

$$v_1 = \sqrt{5gr} \qquad (4.39)$$

The body must have a speed at point 1, v gr 1 ≥ 5 to stay in the circular path. From equations (4.38) and (4.39), it is clear that the minimum speed at the lowest point 1 should be 5 times more than the minimum speed at the highest point 2, so that the body loops without leaving the circle.

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POWER

Power is a measure of how fast or slow a work is done. Power is defined as the rate of work done or energy delivered.

$$\text{Power } (P) = \frac{\text{work done}(W)}{\text{time taken}(t)}$$

$$P = \frac{W}{t}$$

Average power

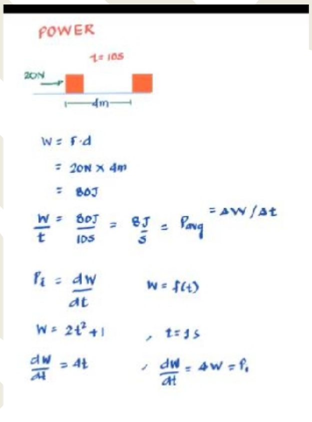
The average power (Pav) is defined as the ratio of the total work done to the total time taken.

$$P_{av} = \frac{\text{total work done}}{\text{total time taken}}$$

Instantaneous power

The instantaneous power (Pinst) is defined as the power delivered at an instant (as time interval approaches zero),

$$P_{inst} = \frac{dW}{dt}$$



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