UNIT 10 OSCILLATIONS

Warm greetings:

The previous notes and videos uploaded are very useful to you. Now we are going to discuss about

- **Periodic motion**
- **Non periodic motion**
- **Oscillatory motion**
- **Simple harmonic motion(SHM)**

Introduction:

❖ Have you seen the Thanjavur Dancing Doll (In Tamil, it is called 'Thanjavur thalayatti bommai')?. It is a world famous Indian cultural doll (Figure 10.1).

❖ What does this Doll do when disturbed? It will dance such that the head and body move continuously in a to and fro motion, until the movement gradually stops.

❖ Similarly, when we walk on the road, our hands and legs will move front and back. Again similarly, when a mother swings a cradle to make her child sleep, the cradle is made to move in to and fro motion.

❖ All these motions are different from the motion that we have discussed so far. These motions are shown in Figure 10.2.

❖ Generally, they are known as oscillatory motion or vibratory motion. A similar motion occurs even at atomic levels.

❖ When the temperature is raised, the atoms in a solid vibrate about their mean position or equilibrium position.

❖ The study of vibrational motion is very important in engineering applications, such as, designing the structure of building, mechanical equipments, etc.

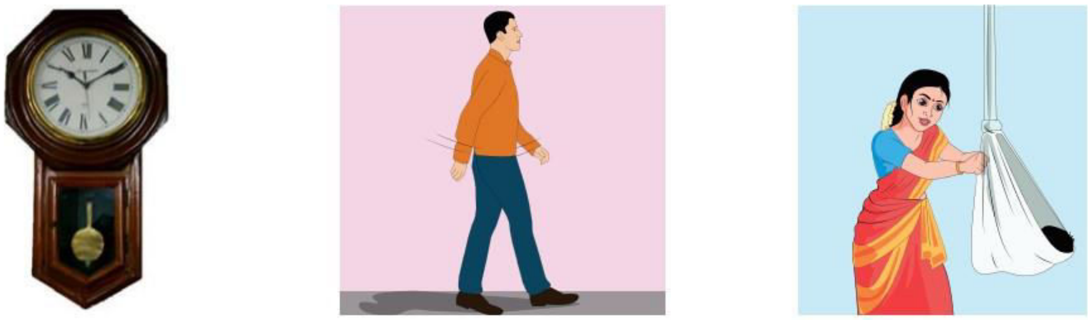


Figure 10.2. Oscillatory or vibratory motions

Periodic and nonperiodic motion:

Motion in physics can be classified as repetitive (periodic motion) and nonrepetitive (non-periodic motion).

1. Periodic motion:

Any motion which repeats itself in a fixed time interval is known as periodic motion.

Examples:

- ✓ Hands in pendulum clock,
- ✓ Swing of a cradle,
- ✓ The revolution of the Earth around the Sun,
- ✓ Waxing and waning of Moon, etc.

2. Non-Periodic motion:

Any motion which does not repeat itself after a regular interval of time is known as non-periodic motion.

Examples:

- ✓ Occurrance of Earth quake,
- ✓ Eruption of volcano.

Oscillatory motion:

- *When an object or a particle moves back and forth repeatedly for some duration of time its motion is said to be oscillatory (or vibratory).*
- Examples; our heart beat,
  swinging motion of the wings of an insect,
  grandfather’s clock (pendulum clock), etc.
- Note that all oscillatory motion are periodic whereas all periodic motions need not be oscillation in nature. See Figure 10.3

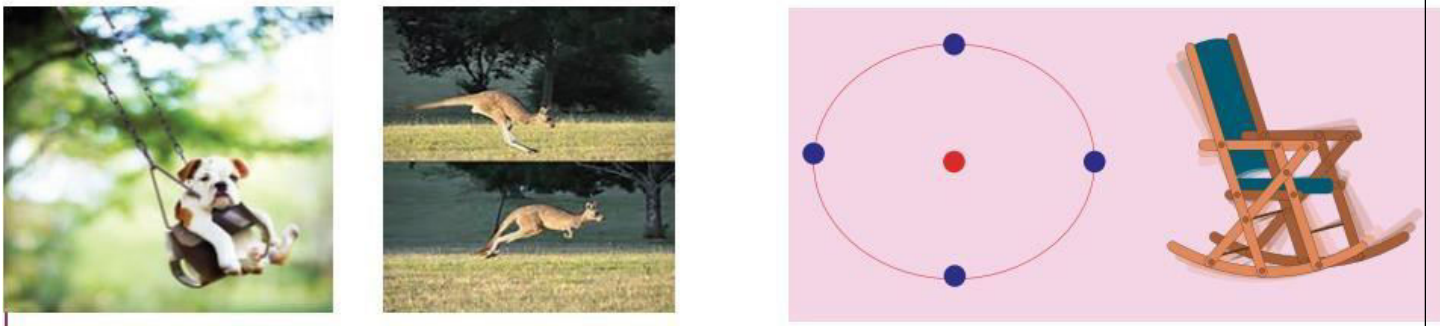


Figure 10.3 Oscillatory or vibratory motions

SIMPLE HARMONIC MOTION (SHM):

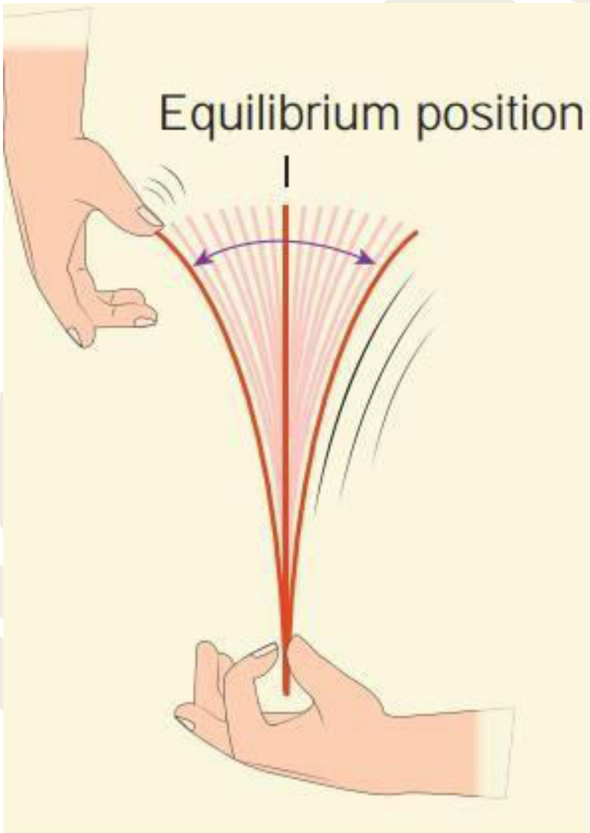


Figure 10.4 Simple Harmonic Motion

- Simple harmonic motion is a special type of oscillatory motion in which the acceleration or force on the particle is directly proportional to its displacement from a fixed point and is always directed towards that fixed point.
- In one dimensional case, let x be the displacement of the particle and a_x be the acceleration of the particle, then

$$a_x \propto x \quad (10.1)$$

$$a_x = -b\,x \quad (10.2)$$

- Where *b* is a constant which measures acceleration per unit displacement and dimensionally it is equal to T^{-2}.
- By multiplying by mass of the particle on both sides of equation (10.2) and from Newton's second law, the force is

$$F_x = -k\,x \quad (10.3)$$

- Where *k* is a force constant which is defined as force per unit length.
- The negative sign indicates that displacement and force (or acceleration) are in opposite directions.
- This means that when the displacement of the particle is taken towards right of equilibrium position (*x* takes positive value), the force (or acceleration) will point towards equilibrium (towards left) and similarly, when the displacement of the particle is taken towards left of equilibrium position (*x* takes negative value), the force (or acceleration) will point towards equilibrium (towards right).
- This type of force is known as ***restoring force*** *because it always directs the particle executing simple harmonic motion to restore to its original (equilibrium or mean) position*.
- This force (restoring force) is central and attractive whose center of attraction is the equilibrium position.
- In order to represent in two or three dimensions, we can write using vector notation

$$\vec{F} = -k\vec{r} \quad (10.4)$$

- Where $\vec{r}$ is the displacement of the particle from the chosen origin. Note that the

force and displacement have a linear relationship. This means that the exponent of force

→ and the exponent of displacement → are unity. The sketch between cause (magnitude of force)|*F* |) and effect (magnitude of displacement |→ |) is a straight line passing through second and fourth quadrant as shown in Figure 10.5. By measuring slope 1/ *k* , one can find the numerical value of force constant *k*.

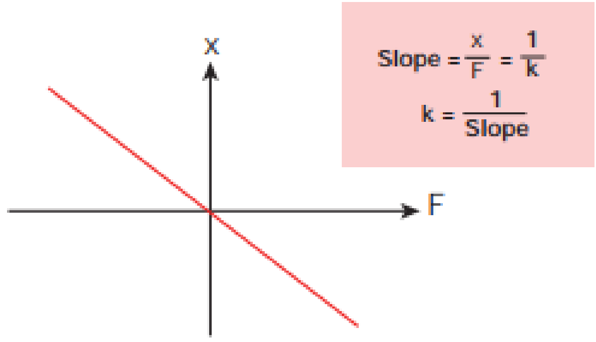


Figure 10.5 Force verses displacement graph

The projection of uniform circular motion on a diameter of SHM:

- Consider a particle of mass *m* moving with uniform speed *v* along the circumference of a circle whose radius is *r* in anti-clockwise direction (as shown in Figure 10.6).
- Let us assume that the origin of the coordinate system coincides with the center *O* of the circle.
- If ω is the angular velocity of the particle and θ the angular displacement of the particle at any instant of time *t*, then $\theta = \omega t$.
- By projecting the uniform circular motion on its diameter gives a simple harmonic motion. This means that we can associate a map (or a relationship) between uniform circular (or revolution) motion to vibratory motion.
- Conversely, any vibratory motion or revolution can be mapped to uniform circular motion. In other words, these two motions are similar in nature.

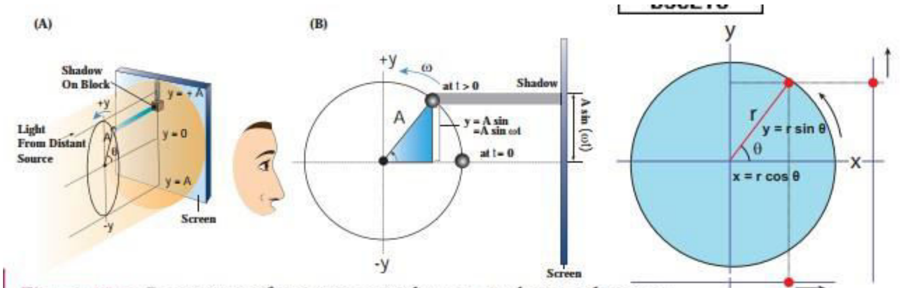


Figure 10.6 Projection of moving particle on a circle on a diameter

- Let us first project the position of particle moving on a circle, on to its vertical diameter or on to a line parallel to vertical diameter as shown in Figure 10.7.
- Similarly, we can do it for horizontal axis or a line parallel to horizontal axis.

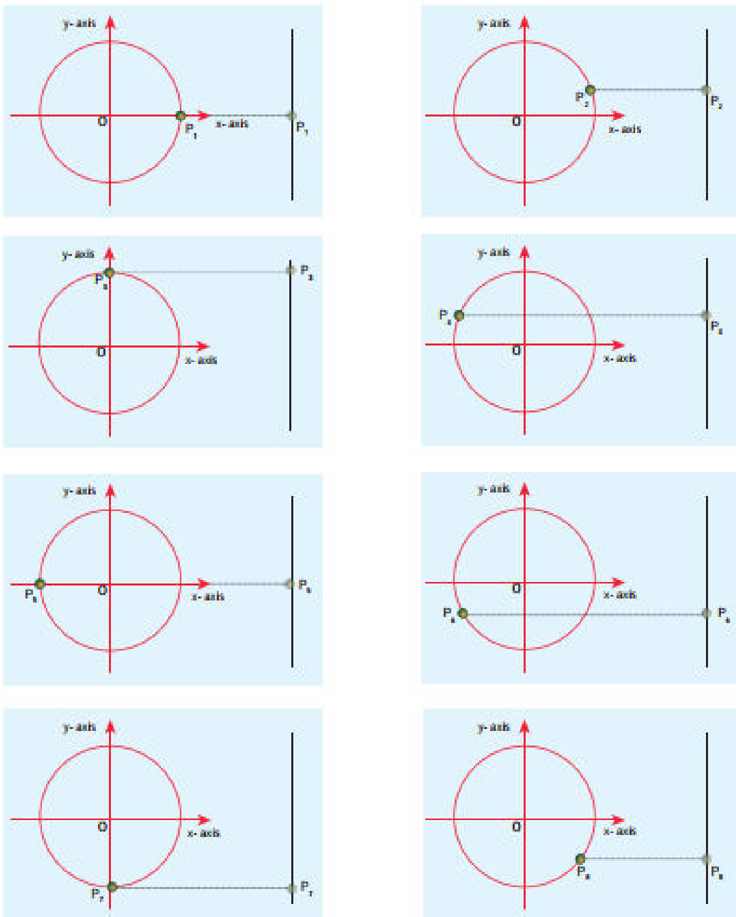


Figure 10.7 The location of a particle at each instant as projected on a vertical axis

As a specific example, consider a spring mass system (or oscillation of pendulum) as shown in Figure 10.8. When the spring moves up and down (or pendulum moves to and fro), the motion of the mass or bob is mapped to points on the circular motion

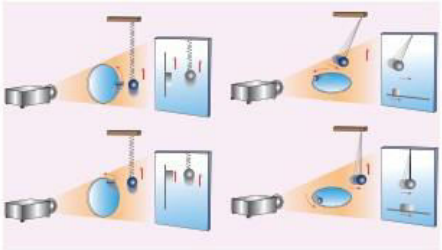


Figure 10.8 Motion of spring mass (or simple pendulum) related to uniform circular motion

Thus, if a particle undergoes uniform circular motion then the projection of the particle on the diameter of the circle (or on a line parallel to the diameter) traces straight-line motion which is simple harmonic in nature. The circle is known as reference circle of the simple harmonic motion. *The simple harmonic motion can also be defined as the motion of the projection of a particle on any diameter of a circle of reference.*

Displacement, velocity, acceleration and its graphical representation – SHM:

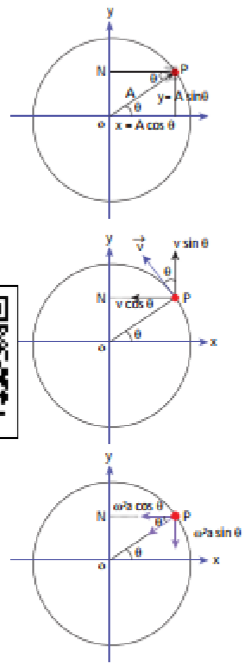


Figure 10.9 Displacement, velocity and acceleration of a particle at some instant of time

The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement. Let *P* be the position of the particle on a circle of radius A at some instant of time *t* as shown in Figure 10.9. Then its displacement y at that instant of time *t* can be derived as follows ;

In ΔOPN

$$\sin\theta = \frac{ON}{OP} \Rightarrow ON = OP\sin\theta \quad (10.5)$$

But $\theta = \omega t$, $ON = y$ and $OP = A$

$$y = A \sin \omega t \quad (10.6)$$

The displacement *y* takes maximum value (which is equal to *A*) when sin ω*t* = 1. This *maximum displacement from the mean position is known as amplitude (A) of the vibrating particle.*

For simple harmonic motion, the amplitude is constant. But, in general, for any motion other than simple harmonic, the amplitude need not be constant, it may vary with time.

Velocity:

The rate of change of displacement is velocity. Taking derivative of equation (10.6) with respect to time, we get

$$v = \frac{dy}{dt} = \frac{d}{dt}(A \sin \omega t)$$

For circular motion (of constant radius), amplitude *A* is a constant and further, for uniform circular motion, angular velocity ω is a constant. Therefore

$$v = \frac{dy}{dt} = A\,\omega \cos \omega t \quad (10.7)$$

Using trigonometry identity,

$$\sin^2 \omega t + \cos^2 \omega t = 1 \Rightarrow \cos \omega t = \sqrt{1-\sin^2 \omega t}$$

we get

$$v = A\omega\sqrt{1-\sin^2 \omega t}$$

From equation (10.6),

$$\sin \omega t = \frac{y}{A}$$

$$v = A\omega\sqrt{1-\left(\frac{y}{A}\right)^2}$$

$$v = \omega\sqrt{A^2 - y^2} \qquad (10.8)$$

From equation (10.8), when the displacement $y = 0$, the velocity $v = \omega A$ (maximum) and for the maximum displacement $y = A$, the velocity $v = 0$ (minimum). As displacement increases from zero to maximum, the velocity decreases from maximum to zero. This is repeated. Since velocity is a vector quantity, equation (10.7) can also be deduced by resolving in to components.

Acceleration:

The rate of change of velocity is acceleration. Taking derivative of equation 10.7 with respect to time,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(A\omega\cos\omega t\right)$$

$$a = -\omega^2 A\sin\omega t = -\omega^2 y \qquad (10.9)$$

$$\therefore \quad a = \frac{d^2y}{dt^2} = -\omega^2 y \qquad (10.10)$$

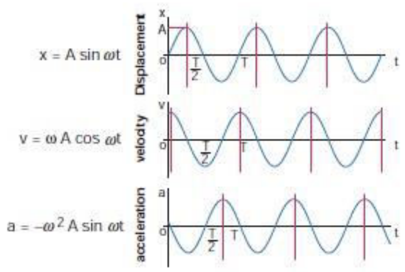


Figure 10.10 Variation of displacement, velocity and acceleration at different instant of time

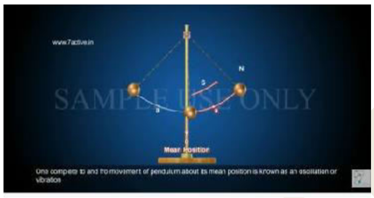
Table 10.1 Displacement, velocity and acceleration at different instant of time.

Time	0	$\frac{T}{4}$	$\frac{2T}{4}$	$\frac{3T}{4}$	$\frac{4T}{4}=T$
ωt	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Displacement $y = A \sin \omega t$	0	A	0	$-A$	0
Velocity $v = A\,\omega \cos \omega t$	$A\,\omega$	0	$-A\,\omega$	0	$A\,\omega$
Acceleration $a = -A\,\omega^2 \sin \omega t$	0	$-A\,\omega^2$	0	$A\,\omega^2$	0

From the Table 10.1 and figure 10.10, we observe that at the mean position (y = 0), velocity of the particle is maximum but the acceleration of the particle is zero. At the extreme position ($y = \pm A$), the velocity of the particle is zero but the acceleration is maximum $A\omega^2$ acting in the opposite direction.

For reference:

https://youtu.be/uM2HpLBVAkA?t=4



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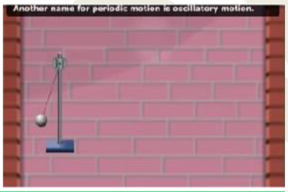

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