UNIT 4
ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Warm greetings:

Dear students

Welcome all. In this section of physics class you get to learn about methods of producing induced emf. We are going to discuss the following topics.

- Mutual induction of solenoids
- Methods of producing induced emf
  - By changing the magnetic field
  - By changing the area A of the coil and
  - By changing the relative orientation θ of the coil with magnetic field

Mutual inductance between two long co-axial solenoids:

Consider two long co-axial solenoids of same length l. The length of these solenoids is large when compared to their radii so that the magnetic field produced inside the solenoids is uniform and the fringing effect at the ends may be ignored. Let A_1 and A_2 be the area of cross section of the solenoids with A_1 being greater than A_2 as shown in Figure 4.22. The turn density of these solenoids are n_1 and n_2 respectively.

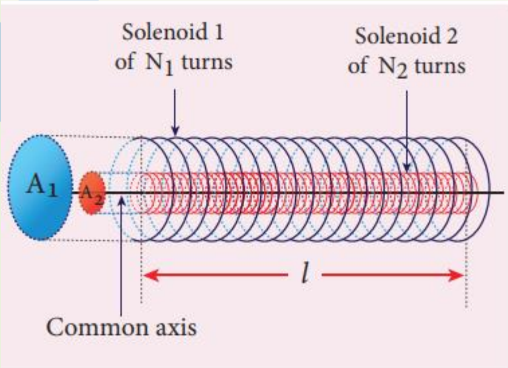

Figure 4.22 Mutual inductance of two long co-axial solenoids

Let i_1 be the current flowing through solenoid 1, then the magnetic field produced inside it is

$$B_1 = \mu_o n_1 i_1$$

As the field lines of B_1 are passing through the area bounded by solenoid 2, the magnetic flux is linked with each turn of solenoid 2 due to current in solenoid 1 and is given by

$$\Phi_{21} = \int_{A_2} \overline{B_1} . d\overline{A} = B_1 A_2 \quad \text{since } \theta = 0^\circ$$
$$= (\mu_o n_1 i_1) A_2$$

The flux linkage with solenoid 2 with total turns N_2 is

$$N_2 \Phi_{21} = (n_2 l)(\mu_o n_1 i_1) A_2 \quad \text{since } N_2 = n_2 l$$
$$N_2 \Phi_{21} = (\mu_o n_1 n_2 A_2 l) i_1$$

We know that $N_2\, \phi_{21} = M_{21} i_1$. Comparing the above equations, we get

$$M_{21} = \mu_o n_1 n_2 A_2 l \qquad (4.13)$$

This gives the expression for mutual inductance M_{21} of the solenoid 2 with respect to solenoid 1. Similarly, we can find mutual inductance M_{12} of solenoid 1 with respect to solenoid 2 as given below. The magnetic field produced by the solenoid 2 when carrying a current i_2 is

$$B_2 = \mu_o n_2 i_2$$

This magnetic field B_2 is uniform inside the solenoid 2 but outside the solenoid 2, it is almost zero. Therefore for solenoid 1, the area A_2 is the effective area over which the magnetic field B_2 is present; not area A_1. Then the magnetic flux Φ_{12} linked with each turn of solenoid 1 due to current in solenoid 2 is

$$\Phi_{12} = \int_{A_2} \overline{B_2} \cdot d\overline{A} = B_2 A_2 = (\mu_o n_2 i_2) A_2$$

The flux linkage of solenoid 1 with total turns N_1 is

$$N_1 \Phi_{12} = (n_1 l)(\mu_o n_2 i_2) A_2 \quad \text{since } N_1 = n_1 l$$

$$N_1\Phi_{12} = (\mu_o n_1 n_2 A_2 l)i_2$$

We know that $N_1\phi_{12} = M_{12}i_2$. Comparing the above equations, we get

$$\therefore M_{12} = \mu_o n_1 n_2 A_2 l \qquad (4.14)$$

From equation (4.22) and (4.23), we can write

$$M_{12} = M_{21} = M \qquad (4.15)$$

In general, the mutual inductance between two long co-axial solenoids is given by

$$M = \mu_o n_1 n_2 A_2 l \qquad (4.16)$$

If a dielectric medium of relative permeability μ_r is present inside the solenoids, then

$$M = \mu n_1 n_2 A_2 l \text{ (or)}$$
$$M = \mu_o \mu_r n_1 n_2 A_2 l$$

METHODS OF PRODUCING INDUCED EMF:

Introduction:

- Electromotive force is the characteristic of any energy source capable of driving electric charge around a circuit.
- We have already learnt that it is not actually a force. It is the work done in moving unit electric charge around the circuit
- . It is measured in J C^{-1} or volt.
- Some examples of energy source which provide emf are electrochemical cells, thermoelectric devices, solar cells and electrical generators.
- Of these, electrical generators are most powerful machines.
- They are used for large scale power generation.
- According to Faraday's law of electromagnetic induction, an emf is induced in a circuit when magnetic flux linked with it changes. This emf is called induced emf.
- The magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} \quad or$$

$$\varepsilon = \frac{d}{dt}(BA\cos\theta) \qquad (4.17)$$

From the above equation, it is clear that induced emf can be produced by changing magnetic flux in any of the following ways.

- By changing the magnetic field
- By changing the area A of the coil and
- By changing the relative orientation θ of the coil with magnetic field.

Production of induced emf by changing the magnetic field:

From Faraday's experiments on electromagnetic induction, it was discovered that an emf is induced in a circuit by changing the magnetic flux of the field through it. The change in flux is brought about by (i) relative motion between the circuit and the magnet (First experiment) (ii) variation in current flowing through the nearby coil (Second experiment).

Production of induced emf by changing the area of the coil:

Consider a conducting rod of length l moving with a velocity υ towards left on a rectangular fixed metallic framework as shown in Figure 4.23. The whole arrangement is placed in a uniform magnetic field $\vec{B}$ whose magnetic lines are perpendicularly directed into the plane of the paper. As the rod moves from AB to DC in a time dt, the area enclosed by the loop and hence the magnetic flux through the loop decreases.

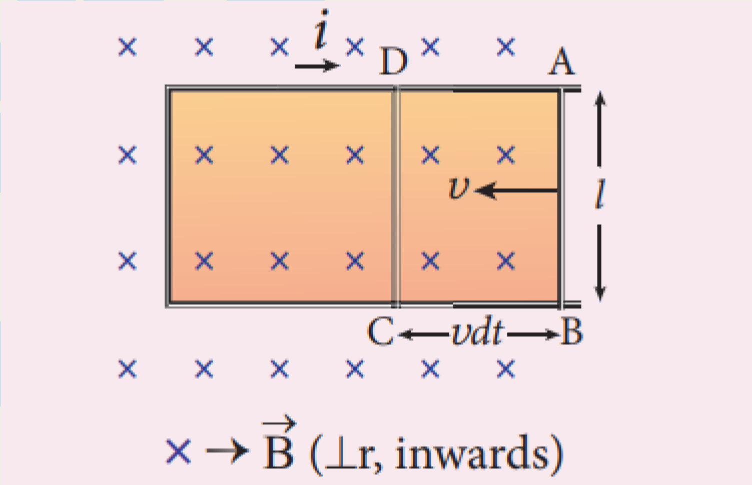

Figure 4.23 Production of induced emf by changing the area enclosed by the loop

The change in magnetic flux in time dt is

$$d\Phi_B = B \times \text{Change in area } (dA)$$
$$= B \times \text{Area } ABCD$$
$$\text{Since Area } ABCD = l(v\,dt)$$
$$d\Phi_B = Blv\,dt \quad \text{(or)}$$
$$\frac{d\Phi_B}{dt} = Blv$$

As a result of change in flux, an emf is generated in the loop. The magnitude of the induced emf is

$$\varepsilon = \frac{d\Phi_B}{dt}$$
$$\varepsilon = Blv \qquad (4.18)$$

This emf is known as **motional emf** since it is produced due to the movement of the conductor in the magnetic field. The direction of induced current is found to be clockwise from Fleming's right hand rule. If R is the resistance of the loop, then the induced current is given by

$$i = \frac{\varepsilon}{R}$$
$$i = \frac{Blv}{R} \qquad (4.19)$$

Energy conservation:

The current-carrying movable rod AB kept in the perpendicular magnetic field experiences a force F_B in the outward direction, opposite to its motion. This force is given by

$$\vec{F}_B = i\,\vec{l} \times \vec{B} = i\,l\,B\sin\theta$$
$$= i\,l\,B \qquad \text{since } \theta = 90°$$

In order to move the rod with a constant velocity $\vec{v}$, a constant force that is equal and opposite to the magnetic force, must be applied.

$$\left|\vec{F}_{app}\right| = \left|\vec{F}_B\right| = i\,l\,B$$

Therefore, mechanical work is done by the applied force to move the rod. The rate of doing work or power is

$$P = \vec{F}_{app} \cdot \vec{v} = F_{app} v \cos\theta \qquad \text{Here } \theta = 0^\circ$$
$$= ilBv$$
$$= \left(\frac{Blv}{R}\right) lBv$$
$$P = \frac{B^2 l^2 v^2}{R} \qquad (4.20)$$

When the induced current flows in the loop, Joule heating takes place. The rate at which thermal energy is dissipated in the loop or power dissipated is

$$P = i^2 R$$
$$P = \left(\frac{Blv}{R}\right)^2 R$$
$$P = \frac{B^2 l^2 v^2}{R} \qquad (4.21)$$

This equation is exactly same as the equation (4.20). Thus the mechanical energy needed to move the rod is converted into electrical energy which then appears as thermal energy in the loop. This energy conversion is consistent with the law of conservation of energy.

Production of induced emf by changing relative orientation of the coil with the magnetic field:

Consider a rectangular coil of *N* turns kept in a uniform magnetic field $\vec{B}$ as shown in Figure 4.24. The coil rotates in anticlockwise direction with an angular velocity *ω* about an axis, perpendicular to the field and to the plane of the paper. At time *t* = 0, the plane of the coil is perpendicular to the field and the flux linked with the coil has its maximum value Φm = *NBA* (where *A* is the area of the coil).

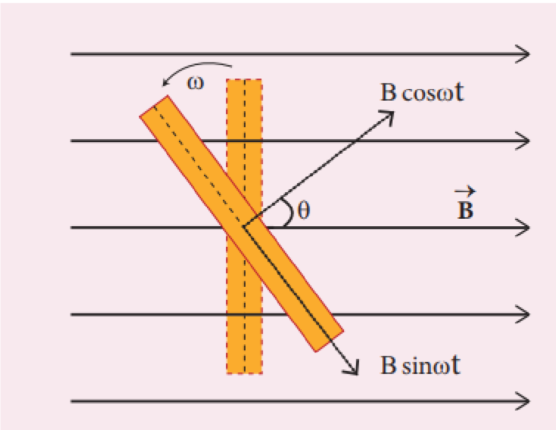

Figure 4.24 The coil has rotated through an angle $\theta = \omega t$

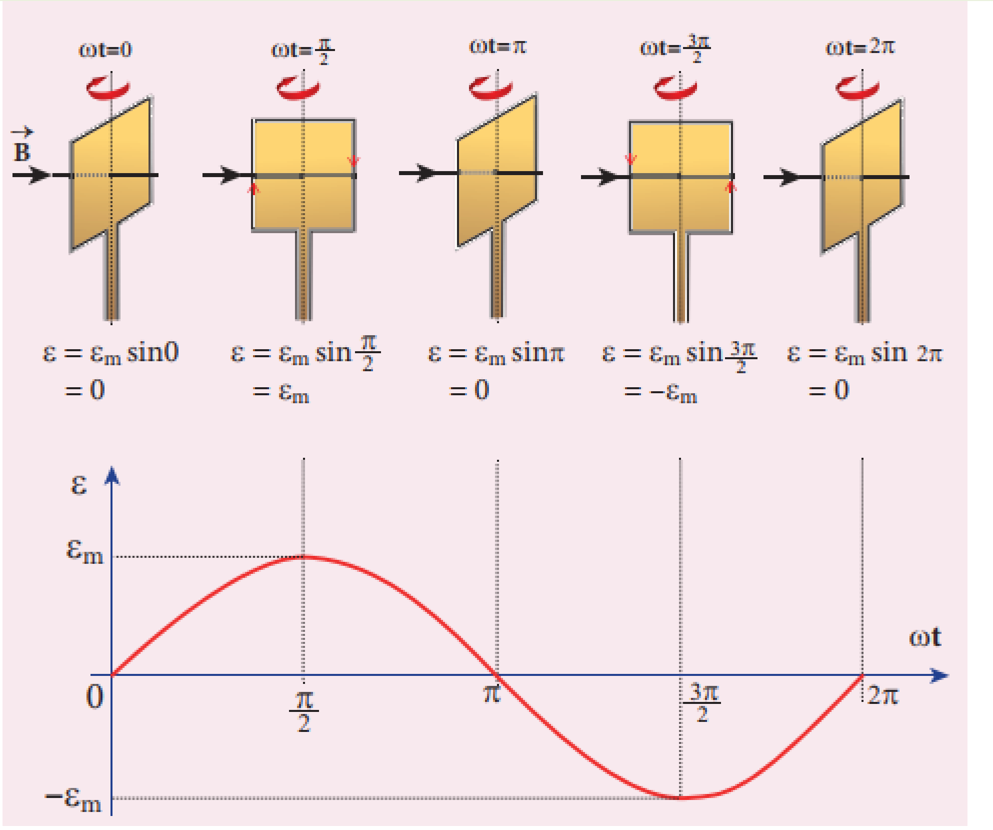

Figure 4.25 Variation of induced emf as a function of ωt

In a time *t* seconds, the coil is rotated through an angle θ (= *ωt*) in anti–clockwise direction. In this position, the flux linked *NBAcosωt* is due to the component of $\vec{B}$ normal to the plane of the coil. The component (*B sinωt*) parallel to the plane has no role in electromagnetic induction. Therefore, the flux linkage with the coil at this deflected position is

$$N\Phi_B = NBA\cos\theta = NBA\cos\omega t$$

According to Faraday's law, the emf induced at that instant is

$$\varepsilon = -\frac{d}{dt}(N\Phi_B) = -\frac{d}{dt}(NBA\cos\omega t)$$
$$= -NBA(-\sin\omega t)\omega$$
$$= NBA\omega\sin\omega t$$

When the coil is rotated through 90o from initial position, sin *ωt* = 1. Then the maximum value of induced emf is

$$\varepsilon_m = NBA\omega$$

Therefore, the value of induced emf at any instant is then given by

$$\varepsilon = \varepsilon_m \sin\omega t \qquad (4.22)$$

It is seen that the induced emf varies as sine function of the time angle ωt. The graph between induced emf and time angle for one rotation of the coil will be a sine curve (Figure 4.25) and the emf varying in this manner is called **sinusoidal emf** or **alternating emf**. If this alternating voltage is given to a closed circuit, a sinusoidally varying current flows in it. This current is called **alternating current** and is given by

$$i = I_m \sin \omega t \quad (4.23)$$

where I_m is the maximum value of induced current.

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