UNIT 4

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Warm greetings:

Dear students

Welcome all. In this section of physics class you get to learn alternating current. We are going to discuss the following topics.

- Sinusoidal AC
- Average of AC
- RMS value of AC
- Phasor

Introduction:

- In section 4.5, we have seen that when the orientation of the coil with the magnetic field is changed, an alternating emf is induced and hence an alternating current flows in the closed circuit.
- **An alternating voltage is the voltage which changes polarity at regular intervals of time and the direction of the resulting alternating current also changes accordingly.**
- In the Figure 4.34(a), an alternating voltage source is connected to a resistor *R* in which the upper terminal of the source is positive and lower terminal negative at an instant. Therefore, the current flows in clockwise direction.
- After a short time, the polarities of the source are reversed so that current now flows in anti-clockwise direction (Figure 4.34(b)). This current which flows in alternate directions in the circuit is called alternating current.

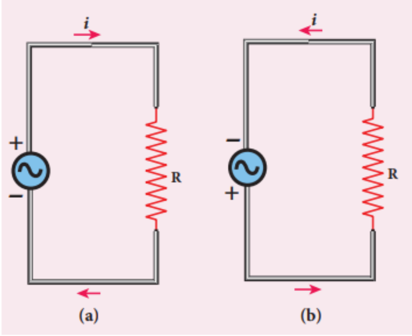

Figure 4.34 Alternating voltage and the corresponding alternating current

Sinusoidal alternating voltage:

If the waveform of alternating voltage is a sine wave, then it is known as sinusoidal alternating voltage which is given by the relation.

$$v = V_m \sin \omega t \quad (4.29)$$

Where υ is the instantaneous value of alternating voltage;

V_m is the maximum value (amplitude) and

ω is the angular frequency of the alternating voltage.

When sinusoidal alternating voltage is applied to a closed circuit, the resulting alternating current is also sinusoidal in nature and its relation is

$$i = I_m \sin \omega t \quad (4.30)$$

where I_m is the maximum value (amplitude) of the alternating current. The direction of sinusoidal voltage or current is reversed after every half-cycle and its magnitude is also changing continuously as shown in Figure 4.35.

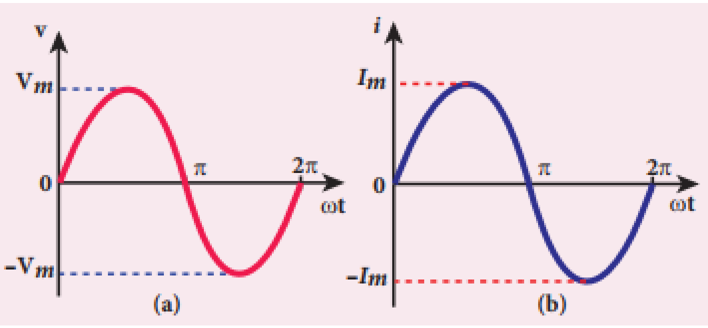

Figure 4.35 (a) Sinusoidal alternating voltage **(b)** Sinusoidal alternating current

Mean or Average value of AC:

- The current and voltage in a DC system remain constant over a period of time so that there is no problem in specifying their magnitudes. However, an alternating current or voltage varies from time to time.
- Then a question arises how to express the magnitude of an alternating current or voltage.
- Though there are many ways of expressing it, we limit our discussion with two ways, namely mean value and RMS (Root Mean Square) value of AC.

Mean or Average value of AC:

- We have learnt that the magnitude of an alternating current in a circuit changes from one instant to other instant and its direction also reverses for every half cycle.
- During positive half cycle, current is taken as positive and during negative cycle it is negative.
- Therefore mean or average value of symmetrical alternating current over one complete cycle is zero.
- Therefore the average or mean value is measured over one half of a cycle. These electrical terms, average current and average voltage, can be used in both AC and DC circuit analysis and calculations.
- **The average value of alternating current is defined as the average of all values of current over a positive half-cycle or a negative half-cycle.**
- The instantaneous value of sinusoidal alternating current is given by the equation

$$i = I_m \sin\omega t \text{ or } i = I_m \sin\theta \quad (\text{where } \theta = \omega t)$$

whose graphical representation is given in Figure 4.36.

- The sum of all currents over a half-cycle is given by area of positive half-cycle (or negative half-cycle). Therefore,

$$I_{av} = \frac{\text{Area of positive half-cycle (or negative half-cycle)}}{\text{Base length of half-cycle}} \qquad (4.31)$$

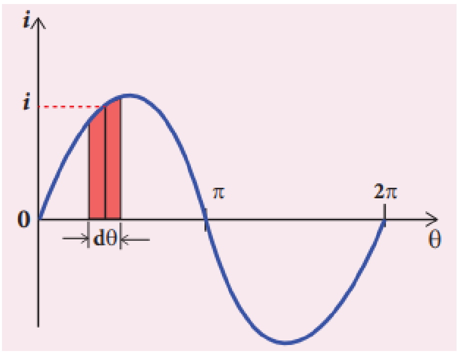

Figure 4.36 Sine wave of an alternating current

Consider an elementary strip of thickness *dθ* in the positive half-cycle of the current wave (Figure 4.41). Let *i* be the mid-ordinate of that strip.

Area of the elementary strip = *i dθ*

Area of positive half-cycle

$$= \int_0^{\pi} i\,d\theta = \int_0^{\pi} I_m \sin\theta\,d\theta$$

$$= I_m\left[-\cos\theta\right]_0^{\pi} = -I_m\left[\cos\pi - \cos 0\right] \; = 2I_m$$

The base length of half-cycle is π. Substituting these values in equation (4.31), we get

$$\text{Average value of AC, } I_{av} = \frac{2I_m}{\pi}$$

$$I_{av} = 0.637\, I_m \qquad (4.32)$$

Hence the average value of AC is 0.637 times the maximum value I_m of the alternating current. For negative half-cycle,

$$I_{av} = -0.637\, I_m.$$

RMS value of AC:

- ✓ The term RMS refers to time-varying sinusoidal currents and voltages which is not used in DC systems.
- ✓ **The root mean square value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle.**
- ✓ It is denoted by I_{RMS}.
- ✓ For alternating voltages, the RMS value is given by V_{RMS}.
- ✓ The alternating current $i = I_m \sin\omega t$ or $i = I_m \sin\theta$ is represented graphically in Figure 4.37.
- ✓ The corresponding squared current wave is also shown by the dotted lines.
- ✓ The sum of the squares of all currents over one cycle is given by the area of one cycle of squared wave.
- ✓ Therefore,

$$I_{RMS} = \sqrt{\frac{\text{Area of one cycle of squared wave}}{\text{Base length of one cycle}}} \quad (4.33)$$

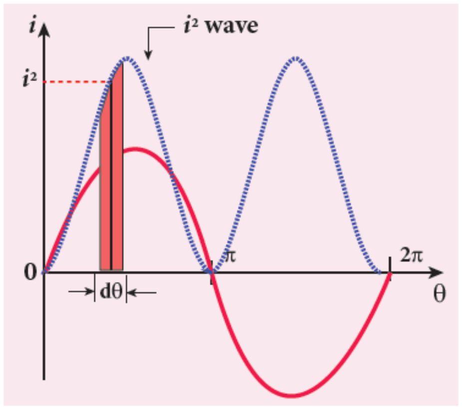

Figure 4.37 Squared wave of AC

An elementary area of thickness dθ is considered in the first half-cycle of the squared current wave as shown in Figure 4.37. Let i^2 be the mid-ordinate of the element.

Area of the element = $i^2 d\theta$

Area of one cycle of squared wave is $= \int_0^{2\pi} i^2 d\theta$

$$= \int_0^{2\pi} I_m^2 \sin^2\theta d\theta = I_m^2 \int_0^{2\pi} \sin^2\theta d\theta \qquad (4.34)$$

$$= I_m^2 \int_0^{2\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$\text{since } \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{I_m^2}{2}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta\right]$$

$$= \frac{I_m^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2}\left[\left(2\pi - \frac{\sin 2\times 2\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{I_m^2}{2} \times 2\pi = I_m^2 \pi \qquad [\because \sin 0 = \sin 4\pi = 0]$$

The base length of one cycle is 2π . Substituting these values in equation (4.33), we get

$$I_{RMS} = \sqrt{\frac{I_m^2 \pi}{2\pi}} = \frac{I_m}{\sqrt{2}}$$

$$I_{rms} = 0.707 I_m \qquad (4.35)$$

Thus we find that for a symmetrical sinusoidal current rms value of current is 70.7 % of its peak value.

Similarly for alternating voltage, it can be shown that

$$V_{rms} = 0.707 V_m \qquad (4.36)$$

Phasor and phasor diagram

Phasor

A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity ω. Such a rotating vector is called a phasor.

A phasor is drawn in such a way that

- ✓ the length of the line segment equals the peak value V_m (or I_m) of the alternating voltage (or current)
- ✓ its angular velocity ω is equal to the angular frequency of the alternating voltage (or current)
- ✓ the projection of phasor on any vertical axis gives the instantaneous value of the alternating voltage (or current)
- ✓ The angle between the phasor and the axis of reference (positive x-axis) indicates the phase of the alternating voltage (or current).
- ✓ The notion of phasors is introduced to analyse phase relationship between voltage and current in different AC circuits.

Phasor diagram:

- ✓ **The diagram which shows various phasors and their phase relations is called phasor diagram**.
- ✓ Consider a sinusoidal alternating voltage $v = V_m \sin \omega t$ applied to a circuit.
- ✓ This voltage can be represented by a phasor, namely $\overrightarrow{OA}$ as shown in Figure 4.38.

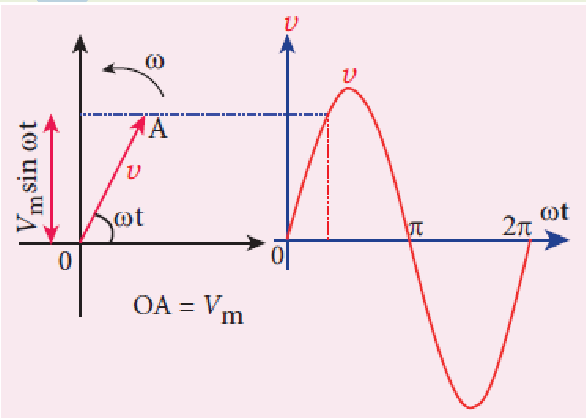

Figure 4.38 Phasor diagram for an alternating voltage $v = V_m \sin \omega t$

- ✓ Here the length of $\overrightarrow{OA}$ equals the peak value (V_m), the angle it makes with x-axis gives the phase (ωt) of the applied voltage. Its projection on y-axis provides the instantaneous value ($V_m \sin \omega t$) at that instant.
- ✓ When $\overrightarrow{OA}$ rotates about O with angular velocity ω in anti-clockwise direction, the waveform of the voltage is generated. For one full rotation of $\overrightarrow{OA}$, one cycle of voltage is produced.
- ✓ The alternating current in the same circuit may be given by the relation $i = I_m \sin(\omega t + \phi)$ which is represented by another phasor $\overrightarrow{OB}$. Here ϕ is the phase angle between voltage and current.
- ✓ In this case, the current leads the voltage by phase angle ϕ which is shown in Figure 4.39. If the current lags behind the voltage, then we write $i = I_m \sin(\omega t - \phi)$.

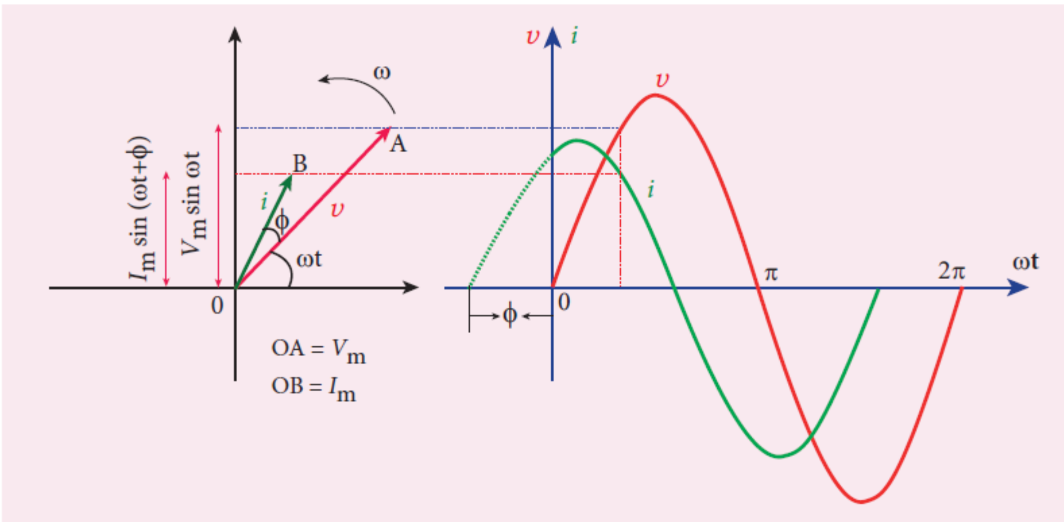

Figure 4.39 Phasor diagram and wave diagram say that i leads v by ϕ

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