UNIT 6 RAY OPTICS

Warm greetings:

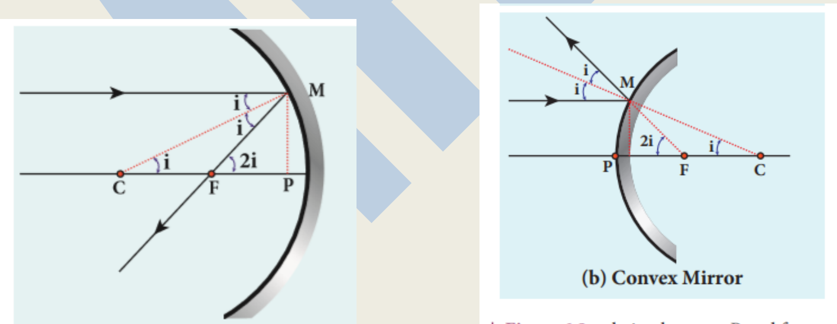
Dear students

Welcome all. In this section of physics class you get to learn about reflection. Now we are going to discuss about the following topics.

- Relation between f & R
- Image formation in spherical mirros
- Cartesian sign convention
- Magnification

Relation between f and R:

Let C be the centre of curvature of the mirror. Consider a light ray parallel to the principal axis is incident on the mirror at M and passes through the principal focus F after reflection. The geometry of reflection of the incident ray is shown in Figure 6.9(a).



(a) Concave mirror

(b) Convex Mirror

Figure 6.9 relation between R and f

The line CM is the normal to the mirror at M. Let i be the angle of incidence and the same will be the angle of reflection.

If MP is the perpendicular from M on the principal axis, then from the geometry,

The angles ∠MCP = i and ∠MFP = 2i

From right angle triangles ΔMCP and ΔMFP,

tan i = PM / PC and tan2i = PM / PF

As the angles are small, tan i ≈ I ,

i = PM / PC and 2i = PM / PF

Simplifying further, 2 PM / PC = PM / PF ; 2PF = PC

PF is focal length f and PC is the radius of curvature R.

$$2f = R \qquad \text{(or)} \qquad f = \frac{R}{2}$$

It is the relation between f and R. The construction is shown for convex mirror in figure 6.9(b)

Image formation in spherical mirrors:

The image can be located by graphical construction. To locate the point of an image, a minimum of two rays must meet at that point. We can use at least any two of the following rays to locate the image point as shown in Figure 6.10.

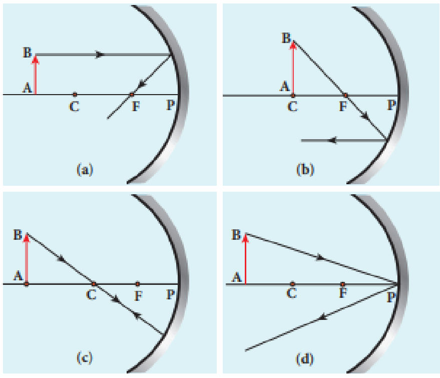


Figure 6.10 Image tracing

1. A ray parallel to the principal axis after reflection will pass through or appear to pass through the principal focus. (Figure 6.10(a))
2. A ray passing through or appear to pass through the principal focus, after reflection will travel parallel to the principal axis. (Figure 6.10(b))
3. A ray passing through the centre of curvature retraces its path after reflection as it is a case of normal incidence.(Figure 6.10(c))
4. A ray falling on the pole will get reflected as per law of reflection keeping principal axis as the normal. (Figure 6.10(d))

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Cartesian sign convention:

While tracing the image, we would normally come across the object distance u, the image distance v, the object height h, the image height (h′), the focal length f and the radius of curvature

R. A system of signs for these quantities must be followed so that the relations connecting them are consistent in all types of physical situations. We shall follow the Cartesian sign convention which is now widely used as given below and also shown in Figure 6.11.

5. The Incident light is taken from left to right (i.e. object on the left of mirror).
6. All the distances are measured from the pole of the mirror (pole is taken as origin).
7. The distances measured to the right of pole along the principal axis are taken as positive.
8. The distances measured to the left of pole along the principal axis are taken as negative.
9. Heights measured in the upward perpendicular direction to the principal axis are taken as positive.
10. Heights measured in the downward perpendicular direction to the principal axis, are taken as negative.

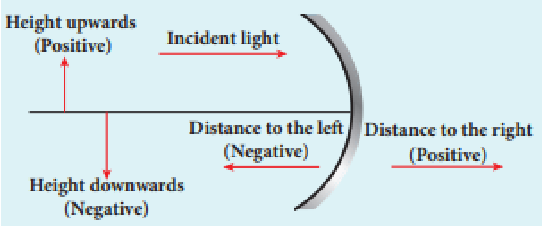
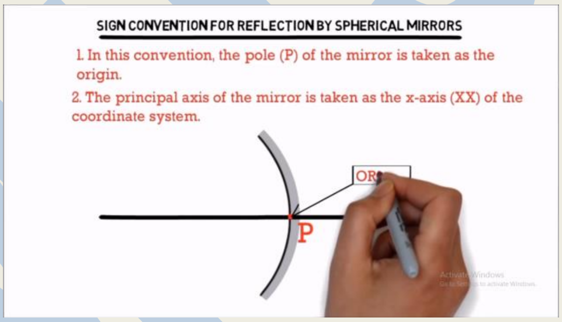


Figure 6.11 Cartesian sign convention



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The mirror equation:

The mirror equation establishes a relation among object distance u, image distance v and focal length f for a spherical mirror. An object AB is considered on the principal axis of a concave mirror beyond the center of curvature C. The image formation is shown in the Figure 6.12.

Let us consider three paraxial rays from point B on the object. The first paraxial ray BD travelling parallel to principal axis is incident on the concave mirror at D, close to the pole P. After reflection the ray passes through the focus F. The second paraxial ray BP incident at the pole P is reflected along PB´. The third paraxial ray BC passing through centre of curvature C, falls normally on the mirror at E is reflected back along the same path. The three reflected rays intersect at the point B ′ . A perpendicular drawn as A ′ B ′ to the principal axis is the real, inverted image of the object AB.

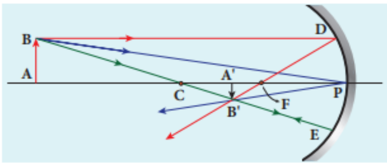


Figure 6.12 Mirror equation

As per law of reflection,

The angle of incidence ∠BPA is equal to the angle of reflection ∠ B P′ ′ A .

The triangles ΔBPA and ΔB P′ A′ are similar. Thus, from the rule of similar triangles,

$$\frac{A'B'}{AB} = \frac{PA'}{PA}$$ ----------------------(A)

The other set of similar triangles are, ΔDPF and Δ B A′ F′ . (PD is almost a straight vertical line)

As, the distances PD = AB the above equation becomes, A ′B ′ / PD = A′ F / PF

$$\frac{A'B'}{AB} = \frac{A'F}{PF}$$ ---------------------------------(B)

From equations (A) & (B) we can write,

PA ′/ PA = A ′F / PF) , the above equation becomes

$$\frac{PA'}{PA} = \frac{PA' - PF}{PF}$$ ----------------------(C)

We can apply the sign conventions for the various distances in the above equation.

PA = −u P A ′ = -v PF = −f

All the three distances are negative as per sign convention, because they are measured to the left of the pole. Now, the equation (C) becomes

$$\frac{-v}{-u} = \frac{-v-(-f)}{-f}$$

On further simplification,

$$\frac{v}{u} = \frac{v-f}{f}; \qquad \frac{v}{u} = \frac{v}{f} - 1$$

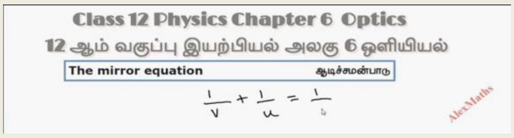
Dividing either side with v,

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

After rearranging,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

The above equation is called mirror equation. Although this equation is derived for a special situation shown in Figure (6.12), it is also valid for all other situations with any spherical mirror. This is because proper sign convention is followed for u, v and f in equation (C).



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Lateral magnification in spherical mirrors:

The lateral or transverse magnification is defined as the ratio of the height of the image to the height of the object.The height of the object and image are measured perpendicular to the principal axis.

Magnification(m) = height of the image (h ′) / height of the object (h)

$$m = h^1 / h$$

Applying proper sign conventions for equation (A),

$$A'B'/AB = PA'/PA$$

$$A'B' = -h, \quad AB = h, \quad PA' = -v, \quad PA = -u$$

$$m = h^1 / h = -v/u$$

Using mirror equation, we can further write the magnification a

$$m = \frac{h'}{h} = \frac{f-v}{f} = \frac{f}{f-u}$$

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