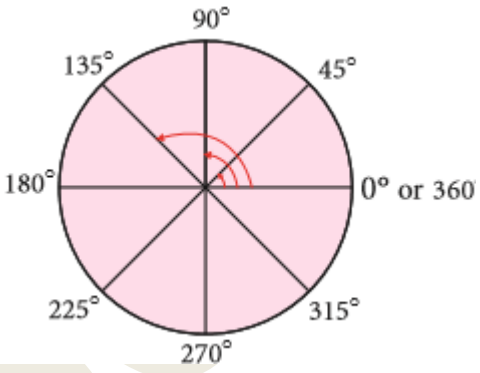
Greeting,

Dear students in the previous notes we learn about Parts of circle and Central angle .Now we are going learn more about Length of the arc and Area of the sector.

Length of the arc and Area of the sector



We have already learnt that a circle of radius 'r' units has

Central angle = 360°

Circumference of the circle = 2πr units

Area of the circle = πr^2 Sq.units.

First, let us take a circle . If a circle is divided into 2 equal sectors we will get 2 semi-circles. The length of a semi-circular arc is half of the circumference of the circle and the area of the semi-circle is half of the area of the circle.

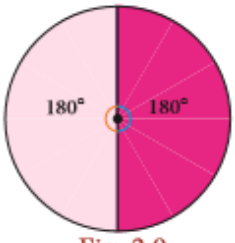


Fig. 2.9

Length of the semi-circular arc $= \frac{1}{2} \times 2\pi r = \frac{180^\circ}{360^\circ} \times 2\pi r$ *units.*

Area of the semi-circle $= \frac{1}{2} \times \pi r^2 = \frac{180^\circ}{360^\circ} \times \pi r^2$ *sq.units.*

Smilarly, if a circle is divided into 3 equal sectors,

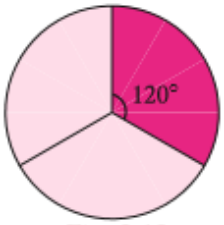
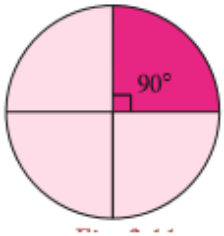


Fig. 2.10

Length of the arc of this sector $= \frac{1}{3} \times 2\pi r = \frac{120^\circ}{360^\circ} \times 2\pi r$ *units*

Area of the sector $= \frac{1}{3} \times \pi r^2 = \frac{120^\circ}{360^\circ} \times \pi r^2$ *sq. units.*

In the same way, if a circle is divided into 4 equal sectors,



Length of the arc of the circular quadrant $= \frac{1}{4} \times 2\pi r = \frac{90^\circ}{360^\circ} \times 2\pi r$ *units.*

Area of the quadrant $= \frac{1}{4} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times \pi r^2$ *sq.units*

What do we know from this?

If the ratio of the central angle of a sector to the central angle of a circle is multiplied with the circumference and the area of the circle, we can find the length of the arc of that sector and its area respectively.

That is, **if we assume that the central angle of a sector of radius 'r' units as θ°,**

then, the ratio of the central angle θ° to 360° is $\frac{\theta^\circ}{360^\circ}$.

$$\text{Length of the arc, } l = \frac{\theta^\circ}{360^\circ} \times 2\pi r \ units$$

$$\text{Area of the sector, } A = \frac{\theta^\circ}{360^\circ} \times \pi r^2 \ sq.units.$$

Note

1. If a circle of radius r units divided into n equal sectors, then the

Length of the arc of each of the sectors $= \frac{1}{n} \times 2\pi r \ units$ and

Area of each of the sectors $= \frac{1}{n} \times \pi r^2 \ sq.units$

2. Also, the area of the sector is derived as

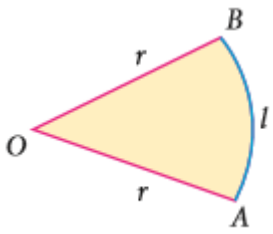
$$= \frac{\theta^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{2}\left(\frac{\theta^\circ}{360^\circ} \times 2\pi r\right) \times r$$

$$= \left(\frac{1}{2} \times l\right) \times r = \frac{lr}{2} \ sq.units$$

Perimeter of a sector

We already know that the total length of the boundary of a closed part is its perimeter, Isn't it? What is the boundary of a sector?



Two radii (OA and OB) and **an arc** $\left(\overset{\frown}{AB}\right)$.

So, the perimeter of a sector = length of the arc + length of two radii

P = lr +2 units

∴ Perimeter of a sector, P = l + 2r units

Tamil people always have a prominent role in the history of Mathematics. They had recorded in the form of a song on how to find the area of a circle, a few thousand years before itself, in the book titled 'Kanakkathikaram'. (கணக்கதிகாரம்). The song is as follows:

"வட்டத்தரை கொண்டு விட்டத்தரை தாக்கச்
சட்டெனத் தோன்றும் குழி "

Meaning:

'வட்டத்தரை' represents half of the circumference and **'விட்டத்தரை'** represents half of the diameter which is the radius and **'குழி'** represents the area. In this song, the Tamil people had recorded that, if half the circumference is multiplied by half the diameter, the area of the circle can be calculated.

Area of the circle = **வட்டத்தரை × விட்டத்தரை**

$$= \frac{1}{2} \text{ of circumference} \times \frac{1}{2} \text{ of diameter} = \left(\frac{1}{2} \times 2\pi r\right) \times r$$

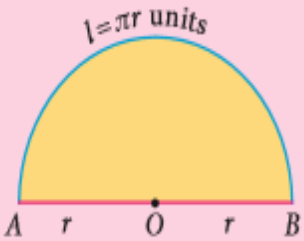
$\therefore$ Area of the circle, $A = \pi r^2$ *sq.units*

Note

1. The perimeter of a semi-circle

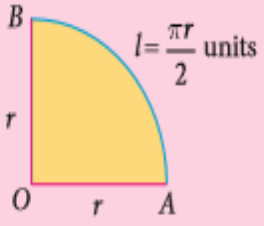
$P = l + 2r$ *units*

$= \pi r + 2r = (\pi + 2)r$ *units*



2. The perimeter of a circular quadrant

$$P = l + 2r = \frac{\pi r}{2} + 2r = \left(\frac{\pi}{2} + 2\right) r \text{ units}$$

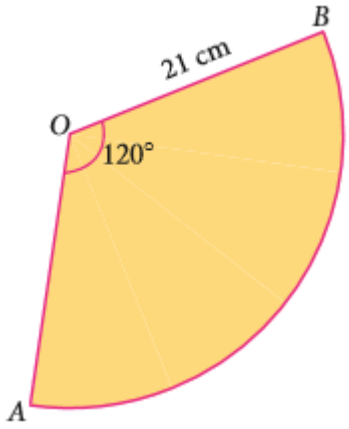


Example:

The radius of a sector is 21cm and its central angle is 120°.

Find (i) the length of the arc (ii) area of the sector

(iii) perimeter of sector ($\pi = \frac{22}{7}$)



Solution:

Radius, r = 21 cm and central angle, θ = 120°.

(i) Length of the arc, $l = \frac{\theta^\circ}{360^\circ}$ X 2π r units

$= \frac{120^\circ}{360^\circ}$ X 2 X $\frac{22}{7}$ X 21

= 44 cm (approximately)

(ii) Area of the sector, $A = \frac{\theta^\circ}{360^\circ}$ X π r ^{2} sq.units.

$= \frac{120^\circ}{360^\circ}$ X $\frac{22}{7}$ X 21 X 21

A = 462 sq.cm (approximately)

Aliter:

Area of the sector

$A = \frac{lr}{2}$ sq.units

$= \frac{44 \times 21}{2} = 462\ cm^2$

(iii) Perimeter of the sector, P = l + 2r units

= 44 + 2 X 21

= 44 + 42

= 86 cm (approximately)

Example 2.1

A circular shaped gymnasium ring of radius 35cm is divided into 5 equal arcs shaded with different colours. Find the length of each of the arcs.

Solution:

Radius, r = 35 cm and n = 5.

Length of each of the arcs, l = $\frac{1}{n}$ X 2π r units

$= \frac{1}{5}$ X 2 x π X 35

= 14 π cm.

Example 2.2

A spinner of radius 7.5 cm is divided into 6 equal sectors. Find the area of each of the sectors.

Solution :

Radius, r = 7.5 cm and n = 6.

Area of each of the sectors , A = $\frac{1}{n}$ X π r ^{2} sq.u

$= \frac{1}{6}$ X π X 7.5 X 7.5

= 9. 375 π sq .cm.

Example 2.3

Kamalesh has a dining table, circular in shape of radius 70 cm whereas Tharun has a circular quadrant dining table of radius 140 cm. Whose dining table has a greater area?

($\pi = \frac{22}{7}$) .

Solution:

Area of the dining table with Kamalesh = π r ^{2} sq. units

Fig. 2.16

$= \frac{22}{7}$ X 70 X 70

A = 15400 sq. cm

Fig. 2.17

Area of the circular quadrant dining table with Tharun

$= \frac{1}{4} \pi r^2$

$= \frac{1}{4}$ X $\frac{22}{7}$ X 140 X 140

A = 15400 sq .cm

We find that, the area of the dining tables of both of them have the same area.

Example 2.4

Four identical medals, each of diameter 7cm are placed as shown in Fig. 2.18.

Find the area of the shaded region between the medals. ($\pi = \frac{22}{7}$) .

Solution :

Diameter, d = 7 cm, therefore r = $\frac{7}{2}$

Area of the shaded region = Area of the square – 4 × Area of the circular quadrant

$= a^2$ X 4 X $\frac{1}{4} \pi r^2$

= (7 X 7) - (4 X $\frac{1}{4}$ X $\frac{22}{7}$ X $\frac{7}{2}$ X $\frac{7}{2}$)

= 49 - 38 .5

= 10 .5 sq.cm.

Formula :

1) Length of the arc, l = $\frac{\theta^\circ}{360^\circ}$. X $2\pi r$ units

2) Area of sector , A = $\frac{\theta^\circ}{360^\circ}$ X πr^2 sq . units

3) Perimeter of a sector, P = l + 2r units

4) Length of the arc of each of the sectors = $\frac{1}{n}$ X $2\pi r$ units

5) Area of each of the sectors = $\frac{1}{n}$ X πr^2 sq . units

6) Central angle $\theta^\circ = \frac{360^\circ}{n}$

Now we enter into exercise problems (4 to 10)

Exercise2.1

Question 4.

For the sectors with given measures, find the length of the arc, area and perimeter.

(π = 3. 14)

(i) central angle 45° r = 16 cm

Answer:

(i) central angle 45° r = 16 cm

Length of the arc l = $\frac{\theta^\circ}{360^\circ} \times 2\pi r$ units

l = $\frac{45^\circ}{360^\circ} \times 2 \times 3.14 \times 16$ cm

l = $\frac{1}{8} \times 2 \times 3.14 \times 16$ cm

l = 12.56 cm

Area of the sector = $\frac{\theta^\circ}{360^\circ} \times \pi r^2$ sq. units

A = $\frac{45^\circ}{360^\circ} \times 3.14 \times 16 \times 16$

A = 100.48 cm^2

Perimeter of the sector P = l + 2r units

P = 12.56 + 2(16) cm

p = 44.56 cm

(ii) central angle 120°, d = 12.6 cm

Answer:

$\therefore r = \frac{12.6}{2}$ cm

r = 6.3cm

Length of the arc $l = \frac{\theta^\circ}{360^\circ} \times 2\pi r$ units

$l = \frac{120^\circ}{360^\circ} \times 2 \times 3.14 \times 63$ cm

l = 13.188cm

I = 13.19cm

Area of the sector $A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$ sq. units

$A = \frac{120^\circ}{360^\circ} \times 3\ 14 \times 6.3 \times 6.3\ cm^2$

$A = 3.14 \times 6.3 \times 2.1\ cm^2$

$A = 41.54\ cm^2$

Perimeter of the sector P = l + 2r cm

P = 13.19 + 2(6.3) cm

= 13.19 + 1.2.6 cm

P = 25.79 cm

Question 5.

From the measures given below, find the area of the sectors.

(i) Length of the arc = 48 m, r = 10 m

Answer:

Area of the sector $A = \frac{lr}{2}$ sq. units

l = 48m

r = 10m

$= \frac{48 \times 10}{2}\ m^2$

$= 24 \times 10m^2$

$= 240\ m^2$

Area of the sector = $240\ m^2$

(ii) length of the arc = 50 cm, r = 13.5 cm

Answer:

Length of the arc l = 12.5 cm

Radius r = 6 cm

Area of the sector A = $\frac{lr}{2}$ sq. units

A = $\frac{12.5\times6}{2}$

A = 12.5 × 3cm^2

A = 37.5 cm^2

Area of the sector A = 37.5 cm^2

Question 6.

Find the central angle of each of the sectors whose measures are given below. ($\pi = \frac{22}{7}$)

(i) area = 462 cm^2, r = 21 cm

Answer:

area = 462 cm^2, r = 21 cm

Radius of the Sector = 21 cm

Area of the sector = 462 cm^2

$\frac{lr}{2}$ = 462

$\frac{l\times21}{2}$ = 462

l = $\frac{462\times2}{21}$

l = 22 × 2

Length of the arc l = 44 cm

$$\frac{\theta^\circ}{360^\circ} \times 2\pi r = 44 \text{ cm}$$

$$\frac{\theta^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21 = 44 \text{ cm}$$

$$\theta^\circ = \frac{44\times360\times7}{2\times22\times21}$$

θ° = 120°

∴ Central angle of the sector = 120°

(ii) length of the arc = 44 m, r = 35 m

Answer:

Radius of the sector = 8.4cm

Area of the sector = 18.48 cm^2

$\frac{lr}{2}$ = 18.48

$\frac{1\times8.4}{2}$ = 18.48

l = $\frac{18.48\times2}{8.4}$

Hint:

$$\frac{{}^{4.4}\cancel{9.24}\ \cancel{18.48}\times\cancel{2}}{\cancel{8.4}\ \cancel{4.2}\ \cancel{2.1}_{1}}$$

Length of the arc l = 4.4 cm

$$\frac{\theta^\circ}{360^\circ}\times 2\pi r = 4.4\text{ cm}$$

$$\frac{\theta^\circ}{360^\circ}\times 2\times\frac{22}{7}\times 8.4 = 4.4\text{ cm}$$

$$\theta^\circ = \frac{4.4\times360\times7}{2\times22\times8.4}$$

Hint:

$$\frac{{}^{1}\cancel{0.1}\ \cancel{0.2}\ \cancel{4.4}\times\cancel{360}\times\cancel{7}}{\cancel{2}_{1}\times\cancel{22}_{1}\times\cancel{8.4}\ \cancel{0.7}\ \cancel{0.1}_{1}}$$

θ° = 30°

Central angle = 30°

Question 7.

A circle of radius 120 m is divided into 8 equal sectors. Find the length of the arc of each of the sectors.

Answer:

Radius of the circle r = 120 m

Number of equal sectors = 8

∴ Central angle of each sector = $\frac{360^\circ}{n}$

θ° = $\frac{360^\circ}{8}$

θ° = 45°

Length of the arc l = $\frac{\theta^\circ}{360^\circ}$ × 2πr units

= $\frac{45^\circ}{360^\circ}$ × 2π × 120 m

Length of the arc = 30 × πm

Another method:

$l = \frac{1}{n} \times 2\pi r = \frac{1}{8} \times 2 \times \pi \times 120 = 30\ \pi$ m

Length of the arc = 30 π m

Question 8.

A circle of radius 70 cm is divided into 5 equal sectors. Find the area of each of the sectors.

Answer:

Radius of the sector r = 70 cm

Number of equal sectors = 5

∴ Central angle of each sector = $\frac{360^\circ}{n}$

$\theta^\circ = 360^\circ$

$\theta^\circ = 72^\circ$

Area of the sector = $\frac{\theta^\circ}{360^\circ} \times \pi r^2$ sq.units

$= \frac{72^\circ}{360^\circ} \times \pi \times 70 \times 70 cm^2$

Hint:

$$\frac{\cancel{72}^{1} \times \pi \times 70 \times \cancel{70}^{14}}{\cancel{360}_{5}}$$

$= 14 \times 70 \times \pi cm^2$

$= 980\ \pi cm^2$

Note: We can solve this problem using $A = \frac{1}{n} \pi r^2$ sq. units also.

Question 9.

Dhamu fixes a square tile of 30cm on the floor. The tile has a sector design on it as shown in the figure. Find the area of the sector. (π = 3.14).

Answer:

Side of the square = 30 cm

∴ Radius of the sector design = 30 cm

Given the design of a circular quadrant.

Area of the quadrant = $\frac{1}{4}$ πr^2 sq.units

= $\frac{1}{4}$ × 3.14 × 30 × 30cm^2

= 3.14 × 15 × 15cm^2

∴ Area of the sector design = 706.5 cm^2 (approximately)

Question 10.

A circle is formed with 8 equal granite stones as shown in the figure each of radius 56 cm and whose central angle is 45°. Find the area of each of the granite stones. (π = $\frac{22}{7}$)

Answer:

Number of equal sectors 'n' = 8

Radius of the sector 'r' = 56 cm

Area of each sector = $\frac{1}{n}$ πr^2 sq. units

= $\frac{1}{8}$ × $\frac{22}{7}$ × 56 × 56cm^2 =1232 cm^2

Area of each sector = 1232 cm^2 (approximately)

Thank you