Greetings !

Dear students in the previous notes we learn the concept of Division of Algebraic expressions, In this notes we will learn the concept of Identities and Application of Identities.

Identities

We have studied in the previous class about standard algebraic identities. An identity is an equation satisfied by any value that replaces its variable(s). Now, we shall recollect four known identities, which are,

$$(a+b)^2 \equiv a^2+2ab+b^2 \qquad (a-b)^2 \equiv a^2-2ab+b^2$$
$$(a^2-b^2) \equiv (a+b)(a-b) \qquad (x+a)(x+b) \equiv x^2+(a+b)x+ab$$

Instead of the symbol $\equiv$, we use $=$ to represent an identity without any confusion.

Example 3.8

Find the value of $(3a + 4c)^2$ by using $(a + b)^2$ identity.

Solution:

Comparing with , we have a = 3a , b = 4c

$$\text{Now } (a+b)^2 = a^2+2ab+b^2$$

$$(3a+4c)^2 = (3a)^2+2(3a)(4c)+(4c)^2 \quad \text{(replacing } a \text{ and } b \text{ values)}$$
$$= 3^2a^2+(2\times3\times4)(a\times c)+4^2c^2$$
$$(3a+4c)^2 = 9a^2+24ac+16c^2$$

Example 3.9

Find the value of 998^2 by using $(a-b)^2$ identity.

Solution:

We know, 998 can be expressed as (1000 − 2)

$$(998)^2 = (1000-2)^2$$

This is in the form of (a – b) ^{2} , we get a = 1000 , b = 2

This is in the form of $(a-b)^2$, we get $a=1000$, $b=2$

Now $(a-b)^2 = a^2 - 2ab + b^2$

$$(1000-2)^2 = (1000)^2 - 2(1000)(2) + (2)^2$$
$$(998)^2 = 1000000 - 4000 + 4 = 996004$$

Example 3.10

Simplify (3x + 5y) (3x – 5y) by using (a + b) (a – b)

Solution:

We have $(3x+5y)(3x-5y)$

Comparing it with $(a+b)(a-b)$ we get $a=3x$ $b=5y$

Now $(a+b)(a-b) = a^2 - b^2$

$$(3x+5y)(3x-5y) = (3x)^2 - (5y)^2 \quad \text{(replacing } a \text{ and } b \text{ values)}$$
$$= 3^2x^2 - 5^2y^2$$
$$(3x+5y)(3x-5y) = 9x^2 - 25y^2$$

Example 3 .11

Expand y ^{2} – 16 by using a^2 – b^2 identity

Solution:

$y^2 - 16$ can be written as $y^2 - 4^2$

Comparing it with $a^2 - b^2$, we get $a = y, b = 4$

Now $a^2 - b^2 = (a+b)(a-b)$

$$y^2 - 4^2 = (y+4)(y-4)$$
$$y^2 - 16 = (y+4)(y-4)$$

Example 3.12

Simplify (5x + 3) (5x + 4) by using (x + a) (x + b) identity.

Solution:

We have $(5x+3)(5x+4)$

Comparing it with $(x+a)(x+b)$, we get $x=5x$ and $a=3, b=4$

We know $(x+a)(x+b)=x^2+(a+b)x+ab$ (replacing x, a and b values)

$$(5x+3)(5x+4) = (5x)^2 + (3+4)(5x) + (3)(4)$$

$$= 5^2x^2+(7)(5x)+12$$

$$(5x+3)(5x+4)=25x^2+35x+12$$

Cubic Identities

I. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

We shall prove it now,

LHS $=(a+b)^3$

$=[(a+b)(a+b)](a+b)$ (expanded form)

$=(a+b)^2(a+b)$

$=(a^2+2ab+b^2)(a+b)$ (using identity)

$=a(a^2+2ab+b^2)+b(a^2+2ab+b^2)$ (using distributive law)

$=a^3+2a^2b+ab^2+ba^2+2ab^2+b^3$

$=a^3+(2a^2b+ba^2)+(ab^2+2ab^2)+b^3$ (grouping 'like' terms)

$=a^3+3a^2b+3ab^2+b^3$

$=RHS$

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$

Hence, we proved the cubic identity by direct multiplication.

Aliter

$\times$	a^2	$2ab$	b^2
a	a^3	$2a^2b$	ab^2
b	a^2b	$2ab^2$	b^3

$=a^3+3a^2b+3ab^2+b^3$

II. $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

We can prove this identity by direct multiplication

We have $(a-b)^3=(a-b)(a-b)(a-b)$

$=(a-b)^2\times(a-b)$

$=(a^2-2ab+b^2)(a-b)$

$=a(a^2-2ab+b^2)-b(a^2-2ab+b^2)$

$=a^3-2a^2b+ab^2-ba^2+2ab^2-b^3$

$=a^3-2a^2b-ba^2+ab^2+2ab^2-b^3$

$=a^3-3a^2b+3ab^2-b^3$

$=RHS$

$(a-b)^3=a^3-3a^2b+3ab^2-b^3$

Hence, we proved.

Aliter

$\times$	a^2	$-2ab$	b^2
a	a^3	$-2a^2b$	ab^2
$-b$	$-a^2b$	$2ab^2$	$-b^3$

$=a^3-3a^2b+3ab^2-b^3$

$a^2b=ba^2$
Multiplication is commutative

III. $(x+a)(x+b)(x+c)=x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$

We know that the identity $(x+a)(x+b)=x^2+(a+b)x+ab$. Let us multiply this by a binomial $(x+c)$. Then we get.

$$\begin{aligned}(x+a)(x+b)(x+c) &= [(x+a)(x+b)](x+c)\\ &= (x^2+(a+b)x+ab)\times(x+c)\\ &= x[x^2+(a+b)x+ab]+c[x^2+(a+b)x+ab] \quad \text{(distributive law)}\\ &= x^3+(a+b)x^2+abx+cx^2+(a+b)xc+abc\\ &= x^3+ax^2+bx^2+abx+cx^2+acx+bcx+abc\\ &= x^3+(a+b+c)x^2+(ab+bc+ca)x+abc \quad (\text{Combine } x^2, x \text{ terms})\end{aligned}$$

$$(x+a)(x+b)(x+c)=x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$$

Thus, we summarise the cubic identities as :

- $(a+b)^3=a^3+3a^2b+3ab^2+b^3$
- $(a-b)^3=a^3-3a^2b+3ab^2-b^3$
- $(x+a)(x+b)(x+c)=x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$

Application of Cubic Identities

I. Using the identity $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

Example 3.13

Expand $(x+4)^3$

Solution:

Comparing $(x+4)^3$ with $(a+b)^3$, we get $a=x, b=4$

We know $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$(4)^2 = 4 \times 4 = 16$

$(4)^3 = 4 \times 4 \times 4 = 64$

$$(x+4)^3=(x)^3+3(x)^2(4)+3(x)(4)^2+(4)^3 \quad \text{(replacing } a, b \text{ values)}$$

$$=(x)^3+3x^2(4)+3(x)(16)+64$$

$$(x+4)^3=x^3+12x^2+48x+64$$

Try to expand this by using

$(a+b)^3=a^3+b^3+3ab(a+b)$

Example 3.14

Find the value of $(103)^3$

Solution:

Now, $(103)^3 = (100+3)^3$

Comparing this with $(a+b)^3$, we get $a=100, b=3$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$ replacing a, b values,

$(100+3)^3 = (100)^3+3(100)^2(3)+3(100)(3)^2+(3)^3$

$= 1000000+3(10000)(3)+3(100)(9)+27$

$= 1000000+90000+2700+27$

$(103)^3 = 1092727$

II. Using the identity $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

Example 3.15

Expand: $(y-5)^3$

Solution:

Comparing $(y-5)^3$ with $(a-b)^3$, we get $a=y, b=5$

$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

$(y-5)^3 = (y)^3-3(y)^2(5)+3(y)(5)^2-(5)^3$

$= (y)^3-3y^2(5)+3(y)(25)-125$

$(y-5)^3 = y^3-15y^2+75y-125$

Try to expand this by using

$(a-b)^3 = a^3-b^3-3ab(a-b)$

Example 3.16

Find the value of $(98)^3$

Solution:

Now, $(98)^3 = (100-2)^3$

Comparing this with $(a-b)^3$, we get $a=100, b=2$

$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

$(100-2)^3 = (100)^3-3(100)^2(2)+3(100)(2)^2-(2)^3$

$= 1000000-3(10000)(2)+3(100)(4)-8$

$= 1000000-60000+1200-8$

$(98)^3 = 941192$

III. Using the identity $(x+a)(x+b)(x+c)=x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$

Example 3.17

Expand: $(x+3)(x+5)(x+2)$

Solution:

Given $(x+3)(x+5)(x+2)$

Comparing this with $(x+a)(x+b)(x+c)$, we get $x=x, a=3, b=5, c=2$

$$(x+a)(x+b)(x+c)=x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$$

$$(x+3)(x+5)(x+2)=(x)^3+(3+5+2)(x)^2+(3\times5+5\times2+2\times3)x+(3)(5)(2)$$

$$=x^3+10x^2+(15+10+6)x+30$$

$$(x+3)(x+5)(x+2)=x^3+10x^2+31x+30$$

Now we are enter into exercise problems

Exercise 3.3

1. Expand

(i) $(3m+5)^2$ (ii) $(5p-1)^2$ (iii) $(2n-1)(2n+3)$ (iv) $4p^2-25q^2$

Answer:

(i) $(3m + 5)^2$

Comparing $(3m + 5)^2$ with $(a + b)^2$ we have $a = 3m$ and $b = 5$

$(a + b)^2 = a^2 + 2ab + b^2$

$(3m + 5)^2 = (3m)^2 + 2(3m)(5) + 5^2$

$= 3^2 m^2 + 30 m + 25$

$= 9m^2 + 30 m + 25$

(ii) $(5p – 1)^2$

Comparing $(5p – 1)^2$ with $(a – b)^2$ we have $a = 5p$ and $b = 1$

$(a – b)^2 = a^2 – 2ab + b^2$

$(5p – 1)^2 = (5p)^2 – 2(5p)(1) + 1^2$

$= 5^2p^2 – 10 p + 1$

$= 25p^2 – 10p + 1$

(iii) $(2n - 1)(2n + 3)$

Comparing $(2n - 1)(2n + 3)$ with $(x + a)(x + b)$ we have $a = -1; b = 3$

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(2n + (-1))(2n + 3) = (2n)^2 + (-1 + 3)2n + (-1)(3)$

$= 2^2 n^2 + 2(2n) - 3 = 4n^2 + 4n - 3$

(iv) $4p^2 - 25q^2 = (2p)^2 - (5q)^2$

Comparing $(2p)^2 - (5q)^2$ with $a^2 - b^2$ we have $a = 2p$ and $b = 5q$

$(a^2 - b^2) = (a + b)(a - b)$

$= (2p + 5q)(2p - 5q)$

Question 2.

Expand

(i) $(3 + m)^3$

(ii) $(2a + 5)^3$

(iii) $(3p + 4q)^3$

(iv) $(52)^3$

(v) $(104)^3$

Answer:

(i) $(3 + m)^3$

Cornparing $(3 + m)^3$ with $(a + b)^3$ we have $a = 3$; $b = m$

$(a + b)^3 = a^2 + 3a^2b + 3ab^2 + b^3$

$(3 + m)^3 = 3^3 + 3(3)^2(m) + 3(3)m^2 + m^3$

$= 27 + 27m + 9m^2 + m^3$

$= m^3 + 9m^2 + 27m + 27$

(ii) $(2a + 5)^3 =$

Comparing $(2a + 5)^3$ with $(a + b)^3$ we have $a = 2a, b = 5$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= (2a)^3 + 3(2a)^2 5 + 3(2a) 5^2 + 5^3$

$= 2^3a^3 + 3(2^2a^2)5 + 6a(25) + 125$

$= 8a^3 + 60a^2 + 150a + 125$

(iii) $(3p + 4q)^3$

Comparing $(3p + 4q)^3$ with $(a + b)^3$ we have $a = 3p$ and $b = 4q$

$(a + b) = a^3 + 3a^2b + 3ab^2 + b^2$

$(3p + 4q)^3 = (3p)^3 + 3(3p)^2(4q) + 3(3p)(4q)^2 + (4q)^3$

$= 3^3p^3 + 3(9p^2)(4q) + 9p(16q^2) - 4^3q^3$

$= 27p^3 + 108p^2q + 144pq^3 + 64q^3$

(iv) $(52)^3$

$(52)^3 = (50 + 2)^3$

Comparing $(50 + 2)^3$ with $(a + b)^3$ we have a = 50 and b =2

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(50 + 2)^3 = 50^3 + 3 (50)^2\, 2 + 3 (50)(2)^2 + 2^3$

$52^3 = 125000 + 6(2,500) + 150(4) + 8$

$= 1,25,000 + 15,000 + 600 + 8$

$52^3 = 1,40,608$

(v) $(104)^3$

$(104)^3 = (100 + 4)^3$

Comparing $(100 + 4)^3$ with $(a + b)^3$ we have a = 100 and b = 4

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(100 + 4)^3 = (100)^3 + 3 (100)^2 (4) + 3 (100) (4)^2 + 4^3$

$= 10,00,000 + 3(10000)4 + 300(16) + 64$

$= 10,00,000 + 1,20,000 + 4,800 + 64 = 11,24,864$

Question 3.

Expand

(i) $(5- x)^3$

(ii) $(2x – 4y)^3$

(iii) $(ab – c)^3$

(iv) $(48)^3$

(v) $(97xy)^3$

Answer:

(i) $(5- x)^3$

Comparing $(5 – x)^3$ with $(a – b)^3$ we have a = 5 and b = x

$(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$

$(5 – x)^3 = 5^3 – 3(5)^2(x) + 3(5)(x^2) – x^3$

$= 125 – 3(25)(x) + 15x^2 – x^3$

$= 125 – 75x + 15 x^2 – x^3$

(ii) $(2x - 4y)^3$

Comparing $(2x - 4y)^3$ with $(a - b)^3$ we have $a = 2x$ and $b = 4y$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(2x - 4y)^3 = (2x)^3 - 3(2x)^2 (4y) + 3(2x) (4y)^2 - (4y)^3$

$= 2^3x^3 - 3(2^2x^2)(4y) + 3(2x) (4^2y^2) - (4^3y^3)$

$= 8x^3 - 48x^2y + 96xy^2 - 64y^3$

(iii) $(ab - c)^3$

Comparing $(ab - c)^3$ with $(a - b)^3$ we have $a = ab$ and $b = c$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(ab - c)^3 = (ab)^3 - 3(ab)^2 c + 3ab (c)^2 - c^3$

$= a^3b^3 - 3(a^2b^2) c + 3abc^2 - c^3$

$= a^3b^3 - 3a^2b^2c + 3abc^2 - c^3$

(iv) $(48)^3$

$(48)^3 = (50 - 2)^3$

Comparing $(50- 2)^3$ with $(a - b)^3$ we have $a = 50$ and $b = 2$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(50 - 2)^3 = (50)^3 - 3(50)^2 (2) + 3 (50)(2)^2 - 2^3$

$= 1,25,000 - 15000 + 600 - 8$

$= 1,10,000 + 592$

$= 1,10,592$

(v) $(97xy)^3$

$(97xy)^3 = 97^3 x^3 y^3 = (100 - 3) x^3y^3$ ... (1)

Comparing$(100 - 3)^3$ with $(a - b)^3$ we have $a = 100, b = 3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(100 - 3)^3 = (100)^3 - 3(100)^2 (3) + 3 (100)(3)^2 - 3^3$

$97^3 = 10,00,000 - 90000 + 2700 - 27$

$97^3 = 910000 + 2673$

$97^3 = 912673$

$97x^3y^3 = 912673x^3y^3$

Question 4.

Simplify (p – 2)(p + 1)(p – 4)

Answer:

(p – 2)(p + 1)(p – 4) = (p + (-2)) (p + 1) (p + (-4))

Comparing (p – 2) (p + 1) (p – 4) with (x + a) (x + b) (x + c) we have x = p ; a = -2; b = 1 ; c = -4.

$(x + a)(x + b)(x + c) = x^2 + (a + b + c) x^2 + (ab + bc+ ca)x + abc$

$= p^3 + (-2 + 1 + (-4)) p^2 + (-2)(1) + (1)(-4) + (-4) (-2))p + (-2) (1) (-4)$

$= p^3 +(-5)p^2 + (-2 + (-4) + 8)p + 8$

$= p^2 – 5p^2 + 2p + 8$

Question 5.

Find the volume of the cube whose side is (x + 1) cm

Answer:

Given side of the cube = (x + 1) cm

Volume of the cube = $(\text{side})^3$ cubic units = $(x + 1)^3$ cm^3

We have $(a + b)^3 = (a3^3 + 3a^2b + 3ab^2 + b^3)$ cm^3

$(x + 1)^3 = (x^3 + 3x^2(1) + 3x(1)^2 + 1^3)cm^3$

Volume = $(x^3 + 3x^2 + 3x + 1)$ cm^3

Question 6.

Find the volume of the cuboid whose dimensions are (x + 2),(x – 1) and (x – 3)

Answer:

Given the dimensions of the cuboid as (x + 2), (x – 1) and (x – 3)

∴ Volume of the cuboid = (l × b × h) $units^3$

= (x + 2) (x – 1) (x – 3) $units^3$

We have $(x + a)(x + b) (x+c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\therefore (x+2) (x- 1) (x-3) = x^3 + (2 – 1 – 3)x^2 + (2 (-1) + (-1) (-3) + (-3) (2)) x + (2) (-1) (-3)$

$= x^3 – 2x^2 + (-2 + 3 – 6)x + 6$

Volume = $x^3 – 2x^3 – 5x + 6$ $units^3$

@@@@@@ Thank you @@@@@@@